Compact Finite Difference Schemes for Interface Problems

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February 04, 2022

Let Γ be a smooth curve inside a two-dimensional rectangular region Ω. Firstly, we propose a highest (sixth) order compact finite difference scheme on uniform Cartesian grids to solve the Poisson interface problems with singular sources along Γ. Using the discrete maximum principle, we provide the proof of the order 6 convergence for the proposed scheme. The coefficient matrix A in the resulting linear system $Ax = b$, following from the proposed scheme, is independent of any source term f, jump conditions, interface curve Γ and boundary conditions. Secondly, we construct a highest (third) order compact finite difference scheme on uniform Cartesian grids for numerically computing both the solution u and the gradient ∇u of the elliptic interface problems with discontinuous and high-contrast coefficients. We prove that the maximum order of the compact finite difference scheme for the elliptic interface problems is three. Finally, we derive a highest (sixth) order compact finite difference scheme for Helmholtz equations with reduced pollution, singular sources and mixed boundary conditions. To reduce the pollution effect, we propose a new pollution minimization strategy that is based on the average truncation error of plane waves. This is joint work with Bin Han, Peter Minev and Michelle Michelle.