

MATH 527 LECTURE 16 EXERCISES AND PROBLEMS

Exercises.

Problems.

Problem 1. Show that

$$u(x, t) \equiv \sum_0^{\infty} \frac{g^{(n)}(t)}{(2n)!} x^{2n} \quad (1)$$

with

$$g(t) \equiv \begin{cases} e^{-\frac{1}{t^k}} & t > 0, k > 1 \\ 0 & t = 0. \end{cases} \quad (2)$$

indeed solves the heat equation.

Problem 2. Show that

$$v(x, t) \equiv u(x, t) - \delta \frac{1}{(4\pi(T + \varepsilon - t))^{n/2}} e^{\left(\frac{|x|^2}{4(T + \varepsilon - t)}\right)}. \quad (3)$$

satisfies $v_t - \Delta v \leq 0$.

Problem*.

Problem. Explore the possibility of proving strong maximum principle through constructing Green's functions. (Note that uniqueness is guaranteed by weak maximum principle)