Math 527 Lecture 7 Exercises and Problems

Exercises.

Exercise 1. Let A, B be $n \times n$ symmetric matrices. Show that

$$A\colon B := \sum_{i,j} A_{ij} B_{ij} = \operatorname{tr}(A\,B). \tag{1}$$

Furthermore show that if A is positive semi-definite, B is negative semi-definite, then $A\colon B \leqslant 0$.

Problems.

Problem 1. (Evans) Let H be uniformly convex with constant θ. Let L be its Legendre transform. Show that L is semi-concave with constant $1/\theta$.

Problem*.