Math 314 Fall 2013 Homework 6

Due Wednesday Oct. 23 5pm in Assignment Box (CAB 3rd Floor)

- There are 6 problems, each 5 points. Total 30 points.
- Please justify all your answers through proof or counterexample.

Question 1. *Let* $f(x) = |x|$. *Prove* ***by definition*** *that* $f(x)$ *is a continuous function (that is* $f(x)$ *is continuous at every* $x_0 \in \mathbb{R}$*).*

Question 2. *Let* $f(x) = \begin{cases} \exp\left[-\frac{1}{x^4}\right] & x \neq 0 \\ 0 & x = 0 \end{cases}$. *Prove (by definition when necessary) that* f *is a continuous function.*

Question 3. *Assume there is* $\delta_0 > 0$ *such that* $h(x) \leqslant f(x) \leqslant g(x)$ *for all* $x \in (x_0 - \delta_0, x_0 + \delta_0)$. *Further assume that* h, g *are continuous at* x_0 *with* $h(x_0) = g(x_0)$. *Prove that* $f(x)$ *is also continuous at* x_0.

Question 4. *Let* $f(x) = x^6 + 5x^5 - 4x^3 + 10x^2 + 7x - 1$. *Prove that there is* $a \in \mathbb{R}$ *such that* $f(a) = 0$.

Question 5. *Let* $A, B \subseteq \mathbb{R}$. *Further assume that there is* $m > 0$ *such that for every* $b \in B$, $|b| < m$. *Let* $C := \{a + b | \, a \in A, b \in B\}$. *Prove that* $\sup A - m \leqslant \sup C \leqslant \sup A + m$.

Question 6. *Let* $\{x_n\}$ *and* $\{y_n\}$ *be sequences of real numbers. Assume* $\lim_{n\to\infty} y_n = 0$. *Prove:*

$$\limsup_{n \longrightarrow \infty} (x_n + y_n) = \limsup_{n \longrightarrow \infty} x_n. \tag{1}$$