MATH 314 FALL 2013 HOMEWORK 2

DUE WEDNESDAY SEPT. 25 5PM IN ASSIGNMENT BOX (CAB 3RD FLOOR)

- There are 6 problems, each 5 points. Total 30 points.
- Please justify all your answers through proof or counterexample.

Question 1. *Let* $E \subseteq \mathbb{R}$. *Prove that* $(E^c)^c = E$.

Question 2. *Let* $A, B \subseteq \mathbb{R}$. *Prove that*

a) $(A \cap B)^c = A^c \cup B^c$;

b) $(A \cup B)^c = A^c \cap B^c$.

Question 3. *Find infinitely many nonempty sets of natural numbers*

$$\mathbb{N} \supset S_1 \supset S_2 \supset \cdots \tag{1}$$

such that $\cap_{n=1}^{\infty} S_n = \varnothing$. *You need to rigorously justify your claim.*

Question 4. *Prove* ***by definition****:*

a) $(0,1) \cup (2,3)$ *is open;*

b) $[0,1] \cup [7,8]$ *is closed.*

Question 5. *Let* $E := \{(-1)^n + e^{-n} : n \in \mathbb{N}\}$. *Find* $\max E, \sup E, \ \min E, \inf E$. *Justify your answers.*

Question 6. *Let* A, $B \subseteq \mathbb{R}$. *Define their sum as the set* $A + B := \{x + y | \ x \in A, \ y \in B\}$. *Prove that* $\sup(A + B) = \sup A + \sup B$, $\inf(A + B) = \inf A + \inf B$.