Math 314 Fall 2013 Homework 10

Due Wednesday Nov. 27 5pm in Assignment Box (CAB 3rd Floor)

- There are 6 problems, each 5 points. Total 30 points.
- Please justify all your answers through proof or counterexample.

Question 1. *Let*

$$f(x) = \begin{cases} 1 & x = 1 \\ 0 & x \neq 1 \end{cases}. \tag{1}$$

Prove by definition that $f(x)$ is Riemann integrable on $[0,2]$.

Question 2. *Let $f(x), g(x)$ be integrable functions on $[a,b]$. Prove by definition that if $f(x) \leqslant g(x)$ for all $x \in [a,b]$, then $\int_a^b f(x)\,\mathrm{d}x \leqslant \int_a^b g(x)\,\mathrm{d}x$.*

Question 3. *Is it true that $|f(x)|$ is integrable on $[a,b] \Longrightarrow f(x)$ is integrable on $[a,b]$? Justify your answer.*

Question 4. *Calculate the following integrals through integration by parts or change of variable.*

$$I_1 = \int_0^\pi e^x \sin x\,\mathrm{d}x; \qquad I_2 = \int_1^e x \ln x\,\mathrm{d}x; \qquad I_3 = \int_1^2 \frac{\mathrm{d}x}{e^x + e^{-x}} \tag{2}$$

Question 5. *Let f be continuous on $[a,b]$. Let $G(x) = \int_{-x}^{\sin x} f(t)\,\mathrm{d}t$. Calculate $G'(x)$. Justify your answer. (Hint: define $F(x) = \int_0^x f(t)\,\mathrm{d}t$ and use F to represent $G(x)$.)*

Question 6. *Prove that the improper integral*

$$\int_0^\infty e^{-2x} \cos(3x)\,\mathrm{d}x \tag{3}$$

exists and calculate its value.