Math 314 Fall 2013 Homework 1

Due Wednesday Sept. 18 5pm in Assignment Box (CAB 3rd Floor)

- There are 6 problems, each 5 points. Total 30 points.
- Please justify all your answers through proof or counterexample.

Question 1. *The Fibonacci numbers are defined through*

$$f_1 = 1, f_2 = 1, f_3 = 2, ... \tag{1}$$

and then through the general formula

$$f_n = f_{n-1} + f_{n-2} \tag{2}$$

for all $n > 2$. Prove using mathematical induction that for all $n > 1$,

$$f_1 + f_3 + \cdots + f_{2n-1} = f_{2n}. \tag{3}$$

Question 2. *Let A, B be mathematical statements. Prove the following*

a) $A \Longrightarrow B$ and $B \Longrightarrow A$ are not equivalent;

b) $A \Longrightarrow B$ and $\neg B \Longrightarrow \neg A$ are equivalent.

Question 3. *Let P be a mathematical statement. If we know that $(\neg P) \Longrightarrow P$ is true, what can we say about P itself?*

Question 4. *Let $P(x), Q(x)$ be statements involving a variable x. Critique the following statement:*

$$(\exists x\ P(x)) \wedge (\exists x\ Q(x)) \Longrightarrow [\exists x\ (P(x) \wedge Q(x))]. \tag{4}$$

If it is true, prove it; If it is false, give a counterexample.

Question 5. *Uniform continuity is defined as follows.*

A real function $f(x)$ is said to be uniformly continuous if

$$\forall \varepsilon > 0\ \exists \delta > 0\ \forall x, y \text{ satisfying } |x - y| < \delta, \qquad |f(x) - f(y)| < \varepsilon. \tag{5}$$

Obtain its working negation "f is not uniformly continuous".

Question 6. *The following are facts:*

A rainy Tuesday is necessary for a rainy Sunday; If Tuesday rains then Wednesday rains. Wednesday rains only if Friday rains. If Monday is sunny then Friday is sunny; A rainy Monday is sufficient for a rainy Saturday.

a) Write the above facts using formal logic statements (Use A — G to denote the statements "Monday rains",...,"Sunday rains".)

b) If we know furthermore that it rains on Sunday. Can we say anything about Saturday? Explain.