Math 314 A1 Fall 2012 Homework 6 Solutions

Due **Thursday** Nov. 22.

5:30pm (Assignment box CAB 3rd floor)

- Related sections in notes: §5.1 – §5.2. Note that improper integral is not covered in this homework.

Problem 1. (2 pts) Are the following functions integrable on $[0,1]$? Justify your answers.

$$f_1(x)=\begin{cases} x^{-1/3} & 0<x\leqslant 1 \\ 0 & x=0 \end{cases}, \qquad f_2(x)=\begin{cases} \dfrac{\sin x}{x} & 0<x\leqslant 1 \\ 1 & x=0 \end{cases}. \tag{1}$$

Problem 2. (3 pts) Let $f(x)$ be integrable on $[a,b]$. Let $c\in\mathbb{R}$. Prove **by definition** that $c\,f(x)$ is integrable and $\int_a^b (c\,f)(x)\,\mathrm{d}x = c\int_a^b f(x)\,\mathrm{d}x$. (Note that you need to discuss the sign of c)

Problem 3. (2 pts) Let $f(x)$, $g(x)$ be integrable functions on $[a,\,b]$. Prove by definition that if $f(x)\leqslant g(x)$ for all $x\in[a,b]$, then $\int_a^b f(x)\,\mathrm{d}x \leqslant \int_a^b g(x)\,\mathrm{d}x$.

Problem 4. (3 pts) Prove that

a) If $f(x)$ is integrable then so is $|f(x)|$

b) and $\left|\int_a^b f(x)\,\mathrm{d}x\right| \leqslant \int_a^b |f(x)|\,\mathrm{d}x$.

c) It is true that $|f(x)|$ is integrable $\Longrightarrow$ $f(x)$ is integrable? Justify your answer.

Problem 5. (6 pts) Calculate the following integrals.

$$I_1=\int_0^\pi e^x\sin x\,\mathrm{d}x; \qquad I_2=\int_1^e x\ln x\,\mathrm{d}x; \qquad I_3=\int_1^2\frac{\mathrm{d}x}{e^x+e^{-x}} \tag{2}$$

Problem 6. (2 pts) Let $m\in\mathbb{N}\cup\{0\}=\{0,1,2,3,\ldots\}$. Calculate

$$I_m=\int_0^{\pi/2}(\sin x)^m\,\mathrm{d}x \text{ and } J_m=\int_0^{\pi/2}(\cos x)^m\,\mathrm{d}x. \tag{3}$$

(Hint: First show $I_m=J_m$ through change of variable. Then apply integration by parts).

Problem 7. (2 pts) Let f be continuous on $[a,b]$. Let $F(x)=\int_{-x}^{2x} f(t)\,\mathrm{d}t$. Calculate $F'(x)$. Justify your answer. (Hint: define $G(x)=\int_0^x f(t)\,\mathrm{d}t$ and use G to represent $F(x)$.)