Math 314 A1 Fall 2012 Homework 3

Due Thursday Oct. 11

5:30pm (Assignment box CAB 3rd floor)

- Related sections in notes: §1.3 – §2.2. Make sure your note is the "updated" version.

Problem 1. (3 pts) If x_n is increasing then there is an extended real number a such that $x_n \longrightarrow a$; If x_n is decreasing then there is an extended real number a such that $x_n \longrightarrow a$.

Problem 2. (4 pts) Let $E := \{(-1)^n + e^{-n} \colon n \in \mathbb{N}\}$. Find $\max E, \sup E, \ \min E, \inf E$. Justify your answers.

Problem 3. (3 pts) Let $A, B \subseteq \mathbb{R}$. Define their sum as the set $A + B := \{x + y |\ x \in A, y \in B\}$. Prove that $\sup (A + B) = \sup A + \sup B$, $\inf (A + B) = \inf A + \inf B$.

Problem 4. (4 pts) Let $\{x_n\}$, $\{y_n\}$ be sequences of real numbers. Which of the following is the most precise relation between $\limsup_{n \longrightarrow \infty} (x_n + y_n)$ and $\limsup_{n \rightarrow \infty} x_n + \limsup_{n \rightarrow \infty} y_n$?

a) $\limsup_{n \rightarrow \infty} (x_n + y_n) = \limsup_{n \rightarrow \infty} x_n + \limsup_{n \rightarrow \infty} y_n$.

b) $\limsup_{n \rightarrow \infty} (x_n + y_n) \leqslant \limsup_{n \rightarrow \infty} x_n + \limsup_{n \rightarrow \infty} y_n$.

c) $\limsup_{n \rightarrow \infty} (x_n + y_n) \leqslant \limsup_{n \rightarrow \infty} x_n + \limsup_{n \rightarrow \infty} y_n$ and it may happen that $\limsup_{n \rightarrow \infty} (x_n + y_n) < \limsup_{n \rightarrow \infty} x_n + \limsup_{n \rightarrow \infty} y_n$.

Justify your answer.

Problem 5. (2 pts) Use interval notation to represent

a) $\{x \in \mathbb{R} \colon x^2 - 3x + 2 > 0\}$.

b) $\left\{x \in \mathbb{R} \colon e^{-x^2} \geqslant \frac{1}{e}\right\}$.

Problem 6. (4 pts) Let $E \subseteq \mathbb{R}$. Prove that E is closed if and only if for every Cauchy sequence $\{x_n\} \subseteq E$, $\lim_{n \longrightarrow \infty} x_n \in E$.