# Math 314 A1 Fall 2012 Homework 1

Due Thursday Sep. 20

5:30pm (Assignment box CAB 3rd floor)

**Problem 1.** Suppose we need to prove uniform boundedness of a function $f(x)$ – that is $|f(x)| \leqslant$ some $M > 0$ for all $x$ - by contradiction, what should the starting assumption be? Explain in formal logic. (Hint: You should first write uniform boundedness into formal expression using $\forall$ and $\exists$)

**Problem 2.** Prove that $x > 0 \Longleftrightarrow x^2 > 0$ is false.

**Problem 3.** Construct the truth table for $(A \,\text{and}\, B) \,\text{or}\, B$ and determine its relation to $B$. Justify your answer.

**Problem 4.** Let $A, B, X$ be sets. Prove $X \setminus (A \cap B) = (X \setminus A) \cup (X \setminus B)$.

**Problem 5.** Find infinitely many nonempty sets of natural numbers

$$\mathbb{N} \supset S_1 \supset S_2 \supset \cdots \tag{1}$$

such that $\cap_{n=1}^{\infty} S_n = \varnothing$. You need to rigorously justify your claim.

**Problem 6.** Let $f: X \mapsto Y$ be a function. Let $A, B \subseteq X$ and $S, T \subseteq Y$. Prove

a) If $A \subseteq B$ then $f(A) \subseteq f(B)$.

b) If $S \subseteq T$ then $f^{-1}(S) \subseteq f^{-1}(T)$.

c) Is it true that $A \subset B$ implies $f(A) \subset f(B)$? Justify your answer.

d) Is it true that $S \subset T$ implies $f^{-1}(S) \subset f^{-1}(T)$? Justify your answer.

**Problem 7.** Let $A \subseteq X, B \subseteq Y$ and $f: X \mapsto Y$. Prove that

a) $f(f^{-1}(B)) \subseteq B$.

b) $f^{-1}(f(A)) \supseteq A$.

c) If $B \subseteq f(X)$, then $f(f^{-1}(B)) = B$.