Math 217 Fall 2013 Homework 8

Due Thursday Nov. 14, 2013 5pm

- This homework consists of 6 problems of 5 points each. The total is 30.
- You need to fully justify your answer for each problem.
- Please read this week's lecture notes before working on the problems.

Question 1. *Let* $A := \{(x, y) \in [0, 1]^2 | x \in \mathbb{Q}, y \notin \mathbb{Q}\}$. *Is* A *Jordan measurable? Justify.*

Question 2. *Let* $A := \left\{\left(\frac{1}{m}, \frac{1}{n}\right) | m, n \in \mathbb{N}\right\}$. *Prove that* $\mu(A) = 0$.

Question 3. *Let* $f(x)$ *be Riemann integrable on* $[a, b]$. *Let* $A := \{(x, f(x)) | x \in [a, b]\}$. *Prove that* $\mu(A) = 0$. *Is the converse –* $\mu(A) = 0 \Longrightarrow f$ *Riemann integrable – true? Justify.*

Question 4. *Let* $f(x) \geqslant 0$ *be Riemann integrable on* $[a, b]$ *and let* $A := \{(x, y) | x \in [a, b], y \leqslant f(x)\}$. *Prove that* A *is Jordan measurable and*

$$\mu(A) = \int_a^b f(x)\, \mathrm{d}x. \tag{1}$$

Is the converse – A *is Jordan measurable* $\Longrightarrow f$ *is Riemann integrable – true? Justify.*

Question 5. *Find a bounded open set that is not Jordan measurable. Justify your answer.*

Question 6. *Prove **by definition** that* $f(x, y) = \sin(x\, y)$ *is Riemann integrable on* $I := [0, 1] \times [0, 1]$.