MATH 217 FALL 2013 HOMEWORK 1

DUE THURSDAY SEPT. 19, 2013 5PM

- This homework consists of 10 problems of 3 points each. The total is 30.
- You need to fully justify your answer – prove that your function indeed has the specified property – for each problem.

Question 1. *Find a bounded sequence* $\{x_n\}$ *that is divergent.*

Question 2. *Find a divergent sequence* $\{x_n\}$ *such that for every* $m \in \mathbb{N}$,

$$\lim_{n \longrightarrow \infty} (x_{n+m} - x_n) = 0. \tag{1}$$

Question 3. *Find a function* $f: \mathbb{R} \mapsto \mathbb{R}$ *that is nowhere continuous, but its absolute value* $|f|$ *is everywhere continuous.*

Question 4. *Find an infinitely differentiable function* f *such that* $\lim_{x \longrightarrow \infty} f(x) = 0$ *holds but* $\lim_{x \longrightarrow \infty} f'(x) = 0$ *does not hold.*

Question 5. *Find a function that is infinitely differentiable (that is* $f^{(n)}$ *exists for all* $n \in \mathbb{N}$*) at every* $x \in \mathbb{R}$ *and satisfy* $f(0) = 1$, $f(x) = 0$ *for all* $|x| \geqslant 1$.

Question 6. *Find a differentiable function* $f: \mathbb{R} \mapsto \mathbb{R}$ *such that* f' *is not continuous.*

Question 7. *Find a differentiable function* $f: \mathbb{R} \mapsto \mathbb{R}$ *such that* $f'(0) > 0$ *but* f *is not increasing on any* (a, b) *containing* 0.

Question 8. *Find a function* $f: [0, 1] \mapsto \mathbb{R}$ *that is bounded on* $[0, 1]$ *but not Riemann integrable.*

Question 9. *Find a function* $f: [0, 1] \mapsto \mathbb{R}$ *such that there is* $F: [0, 1] \mapsto \mathbb{R}$ *such that* $F' = f$, *but* f *is not Riemann integrable on* $[0, 1]$.

Question 10. *Find a function* $f: \mathbb{R} \mapsto \mathbb{R}$ *that is unbounded on every interval* $(a, b) \subseteq \mathbb{R}$. *Recall that a function is bounded on an interval* (a, b) *if there is* $M > 0$ *such that* $\forall x \in (a, b)$, $|f(x)| < M$.