MATH 118 WINTER 2015 FINAL REVIEW

- Final coverage:
  - All lectures and the exercises therein.
  - Homeworks 1 – 8.
- The exercises below are to help you on the concepts and techniques. The exam problems may or may not look like these exercises/problems.
- The exercises below are for materials after Midterm 2. You should also study the reviews for Midterms 1 & 2.

1. Exercises.

Exercise 1. Solve

$$\max/\min f(x) = x^3 - 3x + 3 \quad s.t. \quad -3 \leqslant x \leqslant \frac{3}{2}; \tag{1}$$

$$\max/\min g(x) = \frac{x-1}{x+1} \quad s.t. \quad 0 \leqslant x \leqslant 4; \tag{2}$$

$$\max/\min h(x) = \sin 2x - x \quad s.t. \quad -\frac{\pi}{2} \leqslant x \leqslant \frac{\pi}{2}. \tag{3}$$

(Ans:[1])

Exercise 2. Let f, g be convex on $[a, b]$. Prove that $f + g$ is also convex on $[a, b]$

a) under the assumption that f, g are twice differentiable;

b) under the assumption that f, g are differentiable;

c) under no extra assumptions.

Exercise 3. Find the arc lengths along the following curves.

a) $y = x^{3/2}$ from $x = 0$ to $x = 4$.

b) $y^3 = x^2$ from $(0, 0)$ to $(8, 4)$.

2. More exercises.

Exercise 4. Solve

$$\max f(x) = (1 - x^{2/3})^{3/2} \text{ subject to } -\infty < x < \infty. \tag{4}$$

Exercise 5. Let $f(x): \mathbb{R} \mapsto \mathbb{R}$ be differentiable and such that the only solution to $f'(x) = 0$ is $x = 0$. Further assume that $f''(0) > 0$. Prove or disprove: 0 is the global minimizer.

Exercise 6. Let $a \in \mathbb{R}$. Consider $f(x) := |x|^a$. Find all a such that $f(x)$ is convex on $\mathbb{R}$. Justify.

Exercise 7. Let f, g be convex, nonnegative, and increasing on $[a, b]$. Prove that $f g$ is convex on $[a, b]$.

Exercise 8. Find the volumes of the solids of revolution for $y = 9 - x^2, 0 \leqslant x \leqslant 3$ around the x-axis and y-axis, respectively.

Exercise 9. Find the volume and surface area of the solid generated by revolving the area lying under $y = \sin x$ from $x = 0$ to $x = \pi$ about the x-axis.

3. Problems.

Problem 1. Minimize the surface area of a cylinder when its volume is fixed. (Ans:[2])

Problem 2. Show that of all isoscles triangles inscribed in a given circle, an equilateral triangle has the largest perimeter.

1. $f(-1) = 5, f(-3) = -15$; $g(4) = \frac{3}{5}, g(0) = -1$; $h\left(\mp\frac{\pi}{2}\right) = \pm\frac{\pi}{2}$.
2. $h = 2r$.

Problem 3. Prove or disprove: If f is twice differentiable on $(-1,1)$ and $f''(0)>0$, then there is $\delta>0$ such that f is convex on $(-\delta,\delta)$.

Problem 4. A function $f(x)$ is called Jensen convex (or midpoint convex) on $[a,b]$ if and only if for every x, $y\in[a,b]$, $f\left(\frac{x+y}{2}\right)\leqslant\frac{f(x)+f(y)}{2}$. Prove the following.

i. If $f(x)$ is convex on $[a,b]$ then it is Jensen convex on $[a,b]$;

ii. Assume $f(x)$ is continuous on $[a,b]$. Then f is Jensen convex on $[a,b] \Longrightarrow f$ is convex on $[a,b]$;

Problem 5. Calculate the volume of the solid enclosed by the following surfaces: $x^2+y^2+z^2=3$, $z=0$, $z^2=x^2+y^2-1$.