Math 118 Winter 2015 Homework 8

Due Thursday Mar. 26 3pm in Assignment Box

Question 1. (5 pts) *Solve the following optimization problems (min/max means you need to solve both the minimization and maximization problems)*

a) $\min/\max f(x) = \frac{x^3}{3} - 2x^2 + 3x + 1$ *subject to* $-1 \leqslant x \leqslant 5$;

b) $\min/\max f(x) = -3x^4 + 6x^2 - 1$ *subject to* $-2 \leqslant x \leqslant 2$.

Question 2. (5 pts) *Prove: Among all rectangles inside a fixed circle, the inscribed square has the maximum area and the maximum perimeter.*

Question 3. (5 pts) *Let* $f(x)$ *be infinitely differentiable on* $\mathbb{R}$. *Consider*

$$\min f(x) \qquad \text{subject to } -\infty < x < \infty. \tag{1}$$

a) (2 pts) *Assume* $f'(0) = f''(0) = f'''(0) = 0$ *and* $f^{(4)}(0) > 0$. *What can we conclude about* $x = 0$?
A) local minimizer; B) local maximizer; C) neither; D) not enough information to decide.

b) (3 pts) *Assume* $f'(0) = f''(0) = 0$ *and* $f'''(0) < 0$. *What can we conclude about* $x = 0$?
A) local minimizer; B) local maximizer; C) neither; D) not enough information to decide.

Justify your answers (You should not use results not in our lecture notes).

Question 4. (5 pts) *Let* $f(x)$ *be continuous, strictly increasing on* $[0, a]$ *for some* $a > 0$ *with* $f(0) = 0$. *Let* $g(x)$ *be its inverse function. Prove the following inequality:*

$$\forall x \in [0, a], \quad \forall y \in [0, f(a)], \qquad x\, y \leqslant \int_0^x f(t)\, \mathrm{d}t + \int_0^y g(u)\, \mathrm{d}u. \tag{2}$$

(Hint: Consider $\max F(x) := x\, y - \int_0^x f(t)\, \mathrm{d}t$ *subject to* $0 \leqslant x \leqslant a$.*)*