Math 118 Winter 2015 Midterm Exam 2

Mar. 13, 2015 10am - 10:50am. Total 20+2 Pts

NAME: ID#:

- There are five questions.
- Please write clearly and show enough work.

Question 1. (5 pts) *Is* $\frac{\ln x}{x^2}$ *improperly integrable on* $(1, \infty)$*? Justify your claim.*

Question 2. (5 pts) *Let* $f_n(x) := e^{-nx}\cos(n^2 x)$.

a) **(2 pts)** *Calculate* $\lim_{n\to\infty} f_n(x)$ *on* $(0,\infty)$*;*

b) **(3 pts)** *Is the convergence uniform? Justify your claim.*

Question 3. (5 pts) *Let* $f(x) := \sum_{n=1}^{\infty} \frac{\sin(n^3 x)}{3^n}$.

a) **(1 pt)** *Prove that* $f(x)$ *is defined for all* $x \in \mathbb{R}$.

b) **(2 pts)** *Is* $f(x)$ *continuous on* $\mathbb{R}$*? Justify.*

c) **(2 pts)** *Is* $f(x)$ *differentiable on* $\mathbb{R}$*? Justify.*

Question 4. (5 pts) *Prove that* $\frac{e^{-x} - e^{-2x}}{x}$ *is improperly integrable on* $(0, \infty)$.

Question 5. (Extra 2 pts) *Calculate* $\displaystyle\int_0^\infty \frac{e^{-x}-e^{-2x}}{x}\,\mathrm{d}x.$

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