Math 118 Winter 2015 Homework 7

Due Thursday Mar. 12 3pm in Assignment Box

Question 1. (5 pts) *Let* $a < c < b$. *Assume* $f_n \longrightarrow f$ *uniformly on* $[a, c]$ *and* $[c, b]$. *Prove that* $f_n \rightarrow f$ *uniformly on* $[a, b]$.

Question 2. (5 pts) *Let* $f(x) = \sum_{n=1}^{\infty} \frac{1}{(x+n)^2}$. *Prove that* f *is continuous on* $[0, \infty)$ *and furthermore* $\int_0^1 f(x)\,\mathrm{d}x = 1$.

Question 3. (5 pts) *Let* $\sum_{n=0}^{\infty} a_n x^n$ *and* $\sum_{n=0}^{\infty} b_n x^n$ *be two power series. Assume that there is* $r > 0$ *such that*

$$\forall |x| < r, \qquad \sum_{n=0}^{\infty} a_n x^n = \sum_{n=0}^{\infty} b_n x^n. \tag{1}$$

Prove $\forall n \in \mathbb{N} \cup \{0\}$, $a_n = b_n$.

Question 4. (5 pts) *Let* $f(x) := \sum_{n=1}^{\infty} \frac{\sin(n\,x)}{n^2}$. *Is* $f(x)$ *improperly integrable on* $(0, \infty)$*? Justify.*