Math 118 Winter 2015 Homework 6 Solutions

Due Thursday Mar. 5 3pm in Assignment Box

Question 1. (5 pts) *Calculate the following.*

a) (2 pts) $\lim_{n\to\infty}\frac{1}{1+x^n}$ *on* $\{x|\, x \geqslant 0\}$*;*

b) (3 pts) $\lim_{n\to\infty} e^{-nx}(1+x^2)^n$ *on* $\{x|\, x \geqslant 0\}$.

Question 2. (5 pts) *Let* $f_n(x)$ *converge to* $f(x)$ *uniformly on* $[a,b]$*. Prove that* $\lim_{n\to\infty} f_n(x) = f(x)$ *on* $[a,b]$.

Question 3. (5 pts)

a) (2 pts) *Prove that* $\sum_{n=1}^{\infty} e^{-nx}\sin(n^2 x)$ *converges on* $[0,\infty)$.

b) (3 pts) *Is the convergence uniform? Justify your claim.*

Question 4. (5 pts) *Let* $f_n(x)$, $f(x)$: $[0,\infty) \mapsto \mathbb{R}$*. Further assume* $\lim_{x\to\infty} f_n(x) = 0$ *for every* $n \in \mathbb{N}$ *and* $\lim_{n\to\infty} f_n(x) = f(x)$ *on* $[0,\infty)$.

a) (2 pts) *Does it follow that* $\lim_{x\to\infty} f(x) = 0$*? Justify your claim.*

b) (3 pts) *If instead of convergence we assume* $f_n(x)$ *converges to* $f(x)$ *uniformly on* $[0,\infty)$*, does it follow that* $\lim_{x\to\infty} f(x) = 0$*? Justify your claim.*