MATH 117 FALL 2014 HOMEWORK 9

DUE THURSDAY NOV. 27 3PM IN ASSIGNMENT BOX

QUESTION 1. (5 PTS) *Prove* ***by definition*** *that* $f(x) = \begin{cases} 0 & x \neq 0 \\ 1 & x = 0 \end{cases}$ *is integrable on* $[0, 1]$.

QUESTION 2. (5 PTS) *Let* $f: [a, b] \mapsto \mathbb{R}$ *be integrable. Prove that* $|f(x)|$ *is also integrable on* $[a, b]$ *and furthermore* $\left| \int_a^b f(x)\, \mathrm{d}x \right| \leqslant \int_a^b |f(x)|\, \mathrm{d}x$.

QUESTION 3. (5 PTS) *Let the "Naive Integral" of* $f: [a, b] \mapsto \mathbb{R}$ *be defined as*

$$\mathcal{NI}(f, [a, b]) := \lim_{n \to \infty} \frac{b-a}{n} \sum_{k=1}^{n} f(x_k) \tag{1}$$

where $x_k := a + \frac{k}{n}(b - a)$. *Find real numbers* $a < c < b$ *and a function* $f: [a, b] \mapsto \mathbb{R}$ *such that* $\mathcal{NI}(f, [a, b]) \neq \mathcal{NI}(f, [a, c]) + \mathcal{NI}(f, [c, b])$. *Justify your example and explain why we did not define integrals using (1).*

QUESTION 4. (5 PTS) *Let* $a \in \mathbb{R}$. *Let* $f, g: \mathbb{R} \mapsto \mathbb{R}$ *be such that*

i. f, g *are differentiable on* $\mathbb{R} - \{a\}$;

ii. $\lim_{x \to a} f(x) = +\infty$; $\lim_{x \to a} g(x) = +\infty$;

iii. $\lim_{x \to a} \frac{f'(x)}{g'(x)} = +\infty$.

Prove that $\lim_{x \to a} \frac{f(x)}{g(x)} = +\infty$.