Math 117 Fall 2014 Homework 7

Due Thursday Nov. 6 3pm in Assignment Box

Question 1. (5 pts) *Prove by ε-δ that the Heaviside function* $H(x) := \begin{cases} 1 & x > 0 \\ 0 & x \leqslant 0 \end{cases}$ *is continuous at* $a \neq 0$ *but discontinuous at* $a = 0$.

Question 2. (5 pts) *Let* $f(x) := \begin{cases} e^{-1/x} & x > 0 \\ 0 & x \leqslant 0 \end{cases}$. *Prove that* $f(x)$ *is continuous at every* $a \in \mathbb{R}$.

Question 3. (5 pts) *Prove that the equation* $7x^6 - 9x^5 - 1 = 0$ *has at least two real solutions.*

Question 4. (5 pts) *Let* $f(x) : \mathbb{R} \mapsto \mathbb{R}$ *be such that for every* $s \in \mathbb{R}$, *there are* ***exactly*** *two solutions to* $f(x) = s$. *Prove that* f *is not continuous (we say a function is "continuous" if it is continuous everywhere in its domain).*