Math 117 Fall 2014 Homework 4

Due Thursday Oct. 9 3pm in Assignment Box

Question 1. (10 pts) *Prove the following statements* ***by definition***.

a) (2 pts) $\lim_{n\to\infty}\frac{n!}{n^n}=0$.

b) (2 pts) $\lim_{n\to\infty}\left[\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\cdots+\frac{1}{\sqrt{n^2+n}}\right]=1$.

c) (2 pts) *The sequence* $\left\{(-1)^{n^2}\right\}$ *is divergent.*

d) (2 pts) $\lim_{x\to 0}x^2\sin\left(\frac{1}{x}\right)=0$.

e) (2 pts) *The limit* $\lim_{x\to 0}\sin\left(\frac{1}{x}\right)$ *does not exist.*

Question 2. (5 pts) *A sequence* $\{a_n\}$ *is said to be "bounded above" if and only if there is* $M>0$ *such that* $\forall n\in\mathbb{N}$, $a_n\leqslant M$.

a) (2 pts) *Write down the definition of "*$\{a_n\}$ *is not bounded above", that is write down the working negation of "*$\{a_n\}$ *is bounded above".*

b) (3 pts) *Prove or disprove the following statement:*

If $\{a_n\}$ *is not bounded above, then* $\lim_{n\to\infty}a_n=+\infty$.

Question 3. (5 pts) *Let* $H_n:=1+\frac{1}{2}+\cdots+\frac{1}{n}=\sum_{k=1}^{n}\frac{1}{k}$. *Prove by definition that* $\lim_{n\to\infty}H_n=+\infty$.