Math 117 Fall 2014 Midterm Exam 1

Sept. 26, 2014 10am - 10:50am. Total 20+2 Pts

NAME: ID#:

- There are five questions.
- Please write clearly and show enough work.

Question 1. (5 pts) *Prove that*

a) **(2 pts)** 19 *is prime.*

b) **(3 pts)** $\sqrt{19}$ *is irrational.*

Question 2. (5 pts) *Let* $A := \left\{ \frac{1}{m^2+1} | \, m \in \mathbb{Z} \right\}$. *Calculate* $\inf A$. *Justify.*

Question 3. (5 pts) *Prove that the sequence*

$$\sqrt{2}, \sqrt{2\sqrt{3}}, \sqrt{2\sqrt{3\sqrt{2}}}, \sqrt{2\sqrt{3\sqrt{2\sqrt{3}}}}, ... \tag{1}$$

is increasing and has an upper bound. Then find its limit.

Question 4. (5 pts) *Let A_n be a sequence of sets. Its "limit supreme" is defined as the set*

$$\limsup_{n\to\infty} A_n := \cap_{n\in\mathbb{N}} \cup_{k\in\mathbb{N}, k\geqslant n} A_k. \tag{2}$$

Here $\cup_{k\in\mathbb{N},k\geqslant n}A_k$ means $A_n \cup A_{n+1} \cup A_{n+2} \cup \cdots$. Let $A_n := [0,\, 2 + (-1)^n]$. Calculate $\limsup_{n\to\infty} A_n$. Justify your answer.

Question 5. (Extra 2 pts) *Prove or disprove the following claim:*

$\sqrt{n\,(n+p^2)}$ *is irrational for every* $n \in \mathbb{N}$ *and every* p *prime.*

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