MATH 117 FALL 2014 HOMEWORK 3

DUE THURSDAY OCT. 2 3PM IN ASSIGNMENT BOX

QUESTION 1. (5 PTS) *Let* $B \subseteq Y$ *and* $f: X \mapsto Y$. *Prove that* $f(f^{-1}(B)) \subseteq B$. *Can we replace* $\subseteq$ *by* $=$*? Justify your claim.*

QUESTION 2. (5 PTS) *Prove*

$$\binom{n+d-2}{d-2} + \binom{n+d-3}{d-2} = \binom{n+d-1}{n} - \binom{n+d-3}{n-2} \tag{1}$$

for every pair of natural numbers $n, d \geqslant 2$.

QUESTION 3. (5 PTS) *A function* $f: \mathbb{R} \mapsto \mathbb{R}$ *is said to be "Lipschitz" if the following holds:*

$$\exists M > 0 \quad \forall x, y \in \mathbb{R} \qquad |f(x) - f(y)| \leqslant M\, |x - y|. \tag{2}$$

a) (2 PT) *Find the working negation to (2).*

b) (3 PTS) *Is* $f(x) = x$ *Lipschitz? Is* $g(x) = x^2$ *Lipschitz? Justify your claims.*

QUESTION 4. (5 PTS) *Recall that in the computation of* $\sqrt{2}$, *we have the*

- *Babylonian method:* $a_1 > 0$, $b_1 = 2/a_1$,

$$a_{n+1} = \frac{1}{2}(a_n + b_n), \qquad b_{n+1} = \frac{1}{\frac{1}{2}\left(\frac{1}{a_n} + \frac{1}{b_n}\right)}, \tag{3}$$

- *and Newton's method:* $x_1 > 0$,

$$x_{n+1} = \frac{x_n}{2} + \frac{1}{x_n}. \tag{4}$$

Someone in our class made the conjecture that the two methods are identical. Prove or disprove his conjecture.