The following is a response to Dr. Richelle Marynowski's comments on the draft of new Alberta MATH K-6 curriculum posted at `alberta-curriculum-analysis.ca`.

I read the comments with great interest. I agree with some of them, and disagree with others. Certainly, there are ways to improve the draft. Below, please find my responses to some specific comments.

RM: Looking at some indicated knowledge for students at the Grade 4 level (p. 21) a knowledge statement says "Two or more angles that compose 90° are complementary angles." This is not a correct mathematical definition. Two angles that that make 90° are complimentary, not 2 or more. The same goes for the definition of supplementary angles "Two or more angles that compose 180° are supplementary angles." is incorrect.

VT: Agree. In most cases I have seen the word "complementary" used in mathematics, it applies to a pair of objects. I overlooked this issue (though, I consider it rather minor). I will propose fixing this.

RM: Additionally, on page 21 there is a statement that "Triangles can be classified according to side length as: equilateral; isosceles." Which in itself is not wrong, but what about scalene triangles? Do they not count as triangles? In the Skills and Procedures section, the curriculum states "Classify triangles as equilateral, isosceles, or neither using geometric properties related to sides." Well the 'neither' has a name: it is scalene. And why be so wordy – why can't it be written "Classify triangles as equilateral, isosceles, or scalene according to side length" as this is the only geometric property that is used to classify triangles using these descriptors.

VT: The fact that someone invented a name for a concept does not mean that this concept has to be included in the curriculum. Only important concepts have to be included. What is the significance of scalene triangles? Are there any important facts that are valid specifically for scalene triangles?

I am not aware of a name for quadrilaterals that are neither squares nor rectangles, but I am sure that someone must have given a name to this class of quadrilaterals. Should we include it in the curriculum? What about triangles that are not right? Or isosceles triangles that fail to be equilateral? One can invent millions of concepts, but this does not mean that all these concepts must be included in the curriculum.

RM: In Grade 2, "Pascal's triangle is a triangular arrangement of numbers that illustrates multiple repeating, growing, and symmetrical patterns" (p. 10) is stated under knowledge. Currently the notion of Pascal's triangle is addressed in high school. Having this specific notion of a pattern at this level is completely inappropriate.

VT: While Pascal's triangle is not critical for Grade 2, I see no harm of presenting it there as a simple and neat example. I see nothing inappropriate in it. It is constructed using only addition, so it is totally accessible to Grade 2 students. It indeed contains many interesting patters. We can then revisit it in High School and reveal its connections to more serious mathematics.

RM: Arithmetic and geometric sequences are developed in Grade 4 (p. 25). These are currently also currently in high school. Again, not developmentally appropriate for students in Grade 4 (10 years old) to be working with these kinds of mathematical concepts and patterns.

VT: "Arithmetic sequence" is just a formal term for "skip counting". Are you saying that skip counting is not developmentally appropriate for students in Grade 4? I believe that currently they start it in Grade 1.

RM: In Grade 6, students are adding, subtracting, multiplying, and dividing fractions. Currently, adding/subtracting fractions is in the Grade 7 curriculum with multiplying and dividing being in the Grade 8 curriculum. Time needs to be spent on developing a conceptual understanding of fractions and the different interpretations of fractions (part–whole, ratio, quotient, measure, and operator) before performing operations with fractions. Charalambous and Pitta-Pantazi (2005) commented that "teachers need to scaffold students to develop a profound understanding of the different interpretations of fractions, since such an understanding could also offer to uplift students' performance in tasks related to the operations of fractions" (p. 239). Having a more complete sense of fractions and their uses conceptually before "rushing to provide students with different algorithms to execute operations on fractions" (p. 239) is the opposite of what this draft curriculum proposes.

VT: This comment essentially says that fractions should not be taught too early, but does not say how early is too early. Do you mean that Grade 6 is too early and Grade 7 is not? Are there any studies that suggest that teaching fractions in Grade 6 is too early?

There are studies, however, that support teaching fractions in Grade 6:

"Our main hypothesis was that knowledge of fractions at age 10 would predict algebra knowledge and overall mathematics achievement in high school, above and beyond the effects of general intellectual ability, other mathematical knowledge, and family background. The data supported this hypothesis." (Siegel et al, 2012, p. 693)

"Scores on the fractions comparison test in sixth grade significantly predicted seventh grade mathematics achievement, controlling for central executive span, intelligence, seventh grade fractions comparison performance, and sixth grade mathematics achievement." (Bailey et al, 2012, p. 452)

RM: Pantziara and Philippou (2012) identified that a focus on the part-whole as the primary representation and use of fractions inhibited student development of conceptual understanding of operations with fractions, particularly in problem solving contexts. Ensuring that teachers spend time of the other four ways that fractions are used will help students interpret applications of fractions in context.

VT: I do not see the current draft focus too much on "the part-whole as the primary representation and use of fractions." A considerable portion of the sections on fractions is about operations on fractions.

RM: On page 33 (Grade 6) of the Draft K-6 math curriculum, it states "Division by a fraction can be computed using the formula abcd=abdc." First of all, this is not a formula, it is a procedure. Using this as the way for students to learn division of fractions is very limited and does not develop understanding of what is happening in the process, why it works, and when to use it. The related understanding statement "Division of fractions can be interpreted using multiplication." is also so limited. Sure, it can be, but it can be interpreted using division too.

VT: I presume that "abcd=abdc" was supposed to be $\frac{a}{b} : \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$. The claim that $\frac{a}{b} : \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$ is not a formula is interesting. From my humble point of view as a mathematician, it is a formula. Or an identity, if you wish. This formula may be used as a basis for a procedure. It is common to have procedures based on formulas.

I absolutely agree that, by itself, this formula "does not develop understanding of what is happening in the process". But it is not there by itself; it is a part of a section which should develop the understanding we seek.

RM: This statement: "Multiplication is repeated addition." (p. 33) makes me lose my mind. This statement is written as fact. A more appropriate way to state this is "Multiplication of rational numbers can be represented through repeated addition."

VT: There is no such statement on page 33; not in the version that I have. There is such a statement on page 15, where multiplication of natural numbers is discussed. For natural numbers, this is a perfectly accurate statement. This is exactly how multiplication of natural numbers is defined. Of course, this statement does not extend to rational numbers, but the curriculum does not claim that.

There is a phrase "Multiplication of a natural number by a fraction can be interpreted as repeated addition of the fraction." Again, this is a perfectly accurate statement.

RM: When we use statements like "multiplication is always bigger" or "subtraction always makes smaller" students that have a tenuous hold on mathematical understanding cling to these rules through the rest of their mathematical careers and struggle later on when these rules get broken.

VT: Again, these particular statements that you mention are not in the draft. So your criticism is misplaced.

More generally, every "rule" is only applicable in certain settings, but breaks in other settings. Multiplication is commutative — fails for matrices. The square of a number is always non-negative — fails for complex numbers. There are plenty of examples of this sort. No rule is absolute. Does this mean that we should not teach these rules?

RM: The Draft K-6 curriculum pays lip service to students developing understanding of core mathematical concepts...

VT: Could you please be more specific? What core mathematical concepts receive less attention in the draft curriculum compared to the current one?

In the end of your article, you make a general comment that the new draft goes too fast and many topics are not age appropriate. However, in many other jurisdictions, curriculum progresses faster that what we propose. Do children in those places develop differently? Students in many jurisdictions, including jurisdictions that outperform Alberta, learn fractions by Grade 6. Japan, Singapore, Quebec, France, UK, China, Russia, India, etc, learn fractions by Grade 6. Why can't Alberta children do it?

Reference:

Bailey, D. H. et al, Competence with fractions predicts gains in mathematics achievement. Journal of Experimental Child Psychology, 113(3), 2012, 447–455. doi.org/10.1016/j.jecp.2012.06.004

Siegler, R. S. et al, Early Predictors of High School Mathematics Achievement. Psychological Science, 23(7), 2012, 691–697. doi.org/10.1177/0956797612440101

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