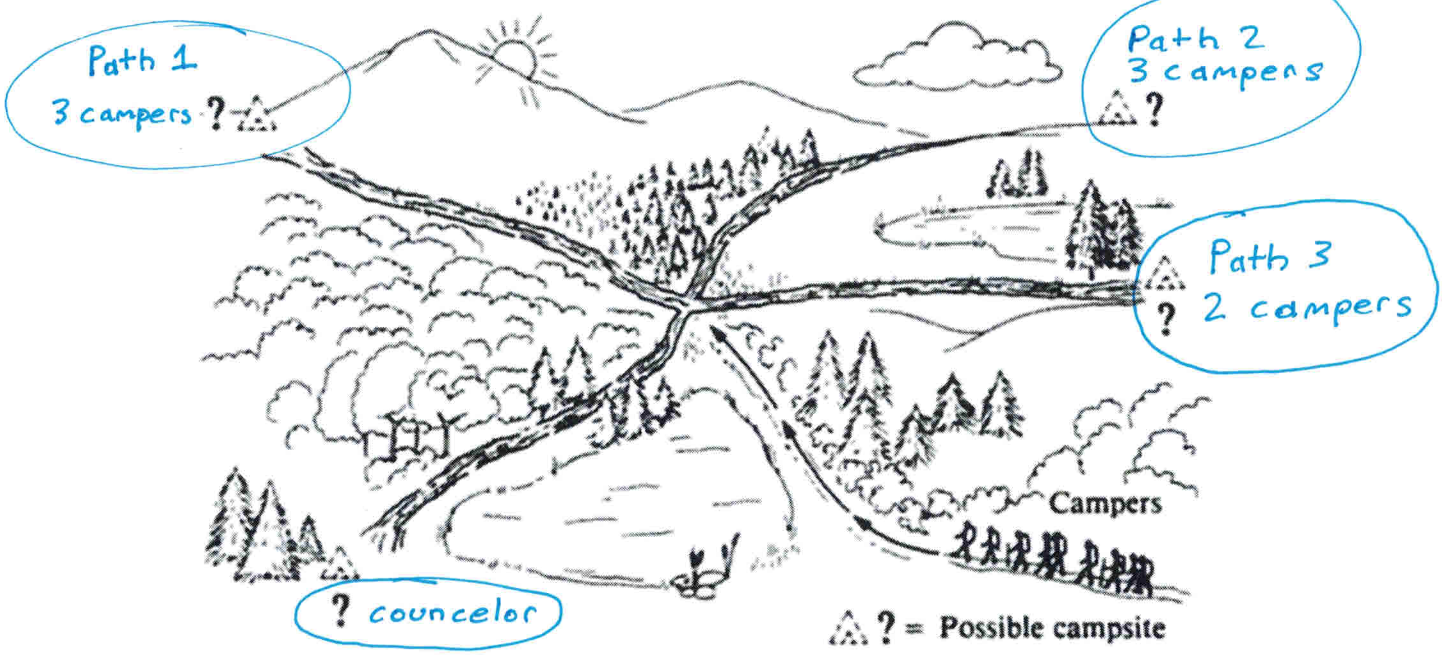
Lecture 7

Warm up problem: **The Campers' Problem** (Section 3.1 in Ecco)

A counsellor has eight campers with her at a junction in a hiking trail. She knows their camp is twenty minutes down one of four possible paths. It will be dark in one hour and the group must find their camp before dark. Two of the eight campers sometimes lie, and unfortunately the counsellor doesn't know which of the eight they are.



Example 1: How can the counselor find the camp?

The councelor checks one path and sends

- 3 campers down path 1
- 3 ——— 2
- 2 ——— 3

CASE 1: There is a disagreement on two of the paths.

- Listen to the majority on paths 1 & 2.

CASE 2: There is a disagreement on at most one path.

- Listen to two paths that agree.

∴ in both cases the councelor knows what is down 3 of the paths and can deduce where the camp is.

Example 2: Design a code that will enable the counselor to accurately deduce the location of the camp.

The counselor can check one path, leaving the other three paths for the 8 campers. She should send 3 campers down each of paths #1 and #2 and send 2 campers down path #3.

To show why this works we will make a code containing three codewords. Each codeword will represent the camp being down a particular path. Each bit in a codeword will represent the answer given by a camper. In this code a 1 will represent "yes" and a 0 will represent "no".

Codeword #1: The camp is down #1 and no one lies:	111	000	00
Codeword #2: The camp is down #2 and no one lies:	000	111	00
Codeword #3: The camp is down #3 and no one lies:	000	000	11

Responses from the 3 campers sent down path

Responses from the 3 campers sent down path

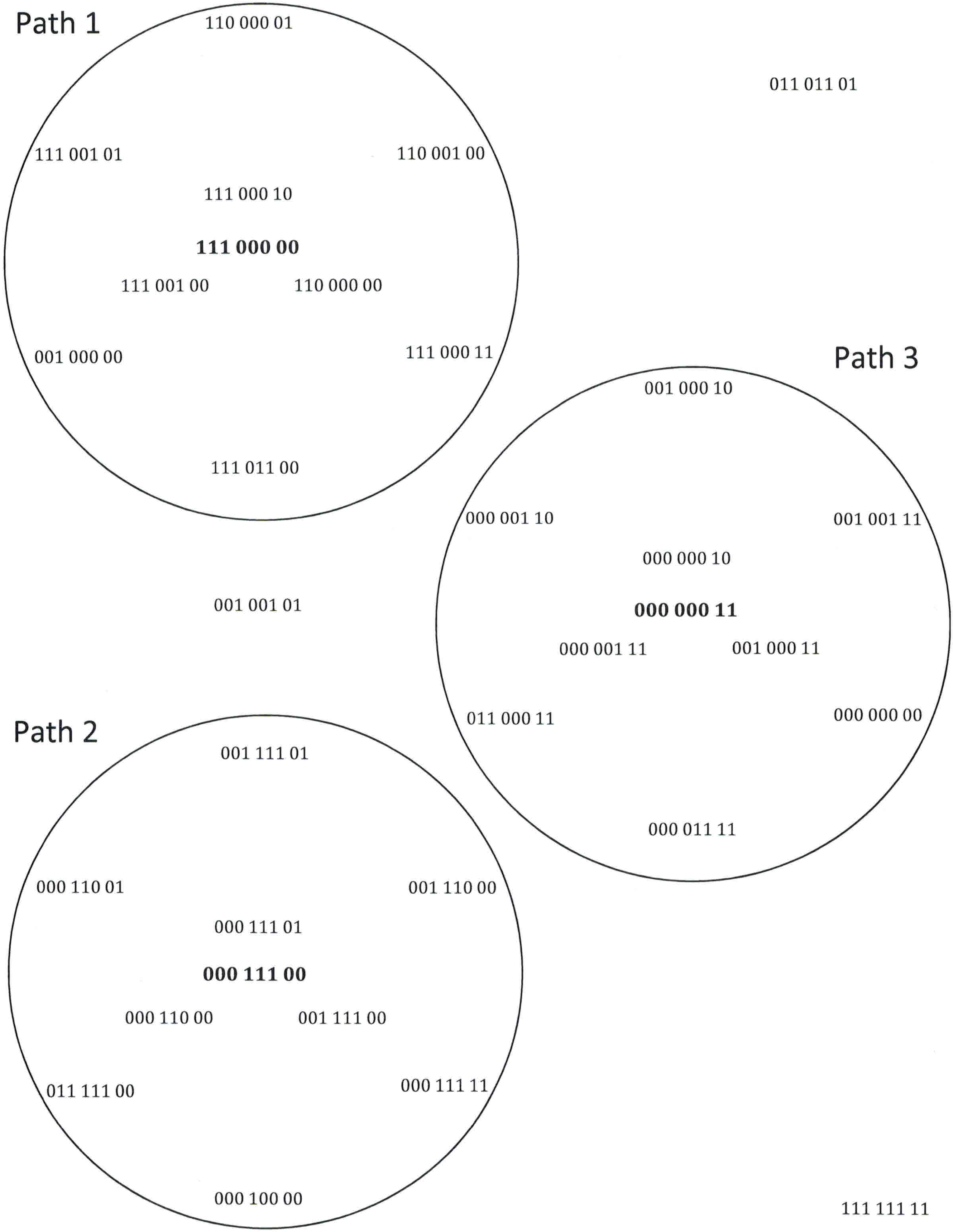
Responses from the 2 campers sent down path #3

Each pair of codewords in the code has a hamming distance of 5 or more. This means the minimum hamming distance of this code is 5. To find out how many errors the code can correct we solve:

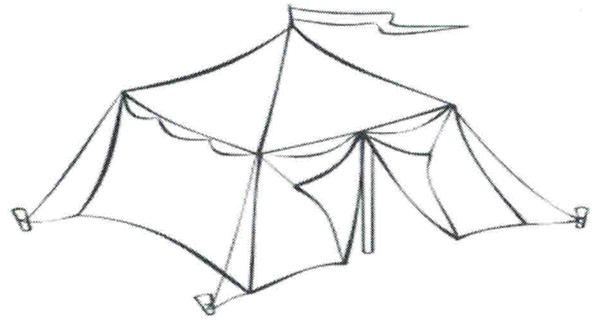
$$2n + 1 = \min\ H(x, y)$$
$$\Rightarrow 2n + 1 = 5$$
$$\Rightarrow n = 2$$

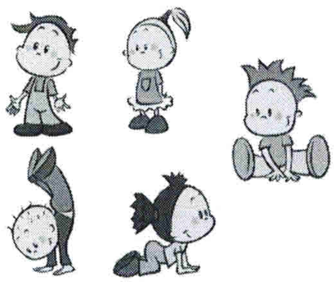
Therefore the code can correct up to 2 corrupted bits, which represent campers that have lied. This means the counselor can correct the two campers that are lying, and therefore can accurately deduce the location of the camp.

Note: The following figure illustrates this solution. In the figure, circles are drawn around each codeword (representing a path). These circles contain erroneous codewords that have a Hamming distance of at most 2 from the real codeword in the center of each circle. Since the circles do not intersect, this code can correct two errors (lying campers). Notice: for clarity, some erroneous codewords have been omitted.

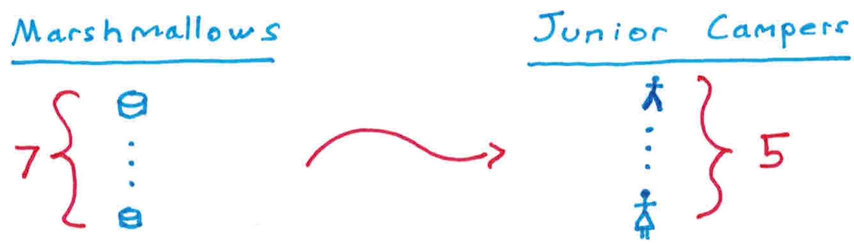


Path 1
110 000 01
111 001 01
110 001 00
111 000 10
111 000 00
111 001 00
110 000 00
001 000 00
111 000 11
111 011 00
011 011 01
Path 3
001 000 10
000 001 10
001 001 11
000 000 10
000 000 11
000 001 11
001 000 11
011 000 11
000 000 00
000 011 11
001 001 01
Path 2
001 111 01
000 110 01
001 110 00
000 111 01
000 111 00
000 110 00
001 111 00
011 111 00
000 111 11
000 100 00
111 111 11





Example 3: Back at camp, 7 mini marshmallows have gone missing. The Camp Director found out that the 7 marshmallows had been stuck up 5 junior camper's noses. Show that there must be a group of 3 junior campers with a combined total of 3 or less marshmallows up their noses.

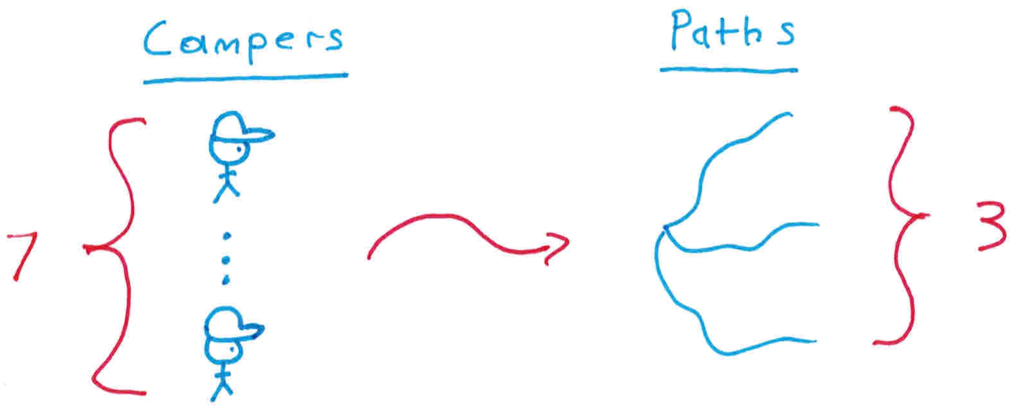


$\therefore$ a junior camper has $\left\lceil \frac{7}{5} \right\rceil = 2$ or more marshmallows in their nose.

$\therefore$ the other four junior campers have a combined total of $7 - \left\lceil \frac{7}{5} \right\rceil = 5$ or less marshmallows up their noses:

$\therefore$ three junior campers have a combined total of $5 - \left\lceil \frac{5}{4} \right\rceil = 3$ or less marshmallows up their noses.

Example 4: Prove the campers' problem cannot be solved with only 7 campers.



$\therefore$ two paths have a combined total of $7 - \left\lceil \frac{7}{3} \right\rceil = 4$ or less campers.

Now when the code from example 2 is used we get:

codeword A :	$11\cdots1$	$0\cdots0$	$0\cdots0$
codeword B :	$00\cdots0$	$1\cdots1$	$0\cdots0$
codeword C :	$00\cdots0$	$0\cdots0$	$1\cdots1$
	3 or more	4 or less	

$\therefore$ $H(\text{codeword } B, \text{codeword } C) \le 4$

$\therefore$ $\min H(x, y) \le 4$

But, to correct 2 corrupted bits we need:

$\min H(x,y) = 2n+1 = 2(2)+1 = 5$ (or more)

$\therefore$ the councelor can no longer correct the two liars.