

§Introduction

Lecture 1

Warm up problem:

- a) There are 8 coins, all identical except that one is counterfeit and is heavier than the others. Show how to find the counterfeit in three weighings using a pan balance.

Can this be done in two weighings?

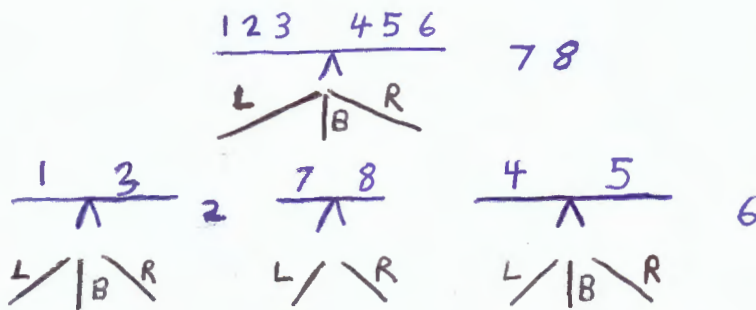
- b) There are 8 coins, all identical except that one is counterfeit and is a different weight than the others. It is not known whether the counterfeit is heavier or lighter. Show how to find the counterfeit in three weighings using a pan balance.

Can this be done in two weighings?

Definition 1: An *adaptive solution* is a step by step solution, where at each step; we adapt depending upon the outcome of previous steps.

Example 1: An adaptive solution to part a)

L - left
R - right
B - balance



counterfeit: 1 2 3 7 8 4 6 5

Result 1: Using n weighings on a pan balance we can find the one counterfeit (because it is heavier than it should be) amongst a maximum of 3^n identical coins

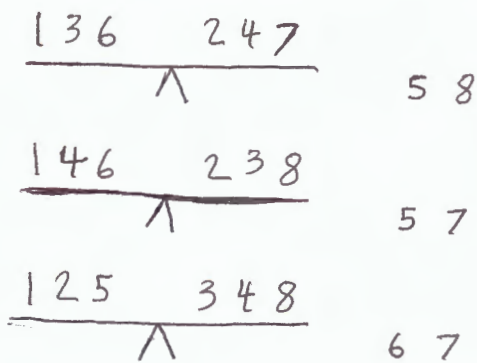
Proof. • Each weighing points to a group: $\frac{\textcircled{1} \quad \textcircled{2}}{\wedge} \quad \textcircled{3}$

• There are 3 groups

• Max of 3^n coins

Definition 2: A nonadaptive solution is a fixed set of steps leading to the solution.

Example 2: An nonadaptive solution to part b)



Complement		
Heavy	Light	counterfeit
LLL OR	RRR	⇒ 1
RRL OR	LLR	⇒ 2
LRR OR	RLL	⇒ 3
RLR OR	LRL	⇒ 4
BBL OR	BBR	⇒ 5
LLB OR	RRB	⇒ 6
RBB OR	LBB	⇒ 7
BRR OR	BLL	⇒ 8

Result 2: Using n weighings on a pan balance it may be possible to find the one counterfeit coin (because it is heavier or lighter than it should be) amongst at most $\lceil \frac{3^n}{2} \rceil$ identical cons.

Proof.

- Each weighing can give 3 results: L, R, B.
- Weighings can spell 3^n distinct words of length n , with letters L, R, B.

Ex. $n=3$:

LLL	RRR
LLR	RRL
LRL	RLR
RLL	LRR
⋮	⋮
BLL	BRR
	BBB

} $3^3 = 27$ words!

- Each word can point to a counterfeit coin but a words complement (flip R's to L's and L's to R's) can point to the same coin.
- $\lceil \frac{3^n}{2} \rceil$ is the max.