NAME: ______________________________________ ID:_______________________

Strong Induction

Consider the recurrence relation defined by:

$$a_0 = 1,$$
$$a_n = a_{n-1} + a_{n-2} + \cdots + a_0 + 1 \qquad \text{for } n \geq 1.$$

Conjecture a solution to this recurrence relation and prove it by strong induction.

Solution:

The first four terms are:

$a_1 = a_0 + 1 = 1 + 1 = 2$

$a_2 = a_1 + a_0 + 1 = 2 + 1 + 1 = 4$

$a_3 = a_2 + a_1 + a_0 + 1 = 4 + 2 + 1 + 1 = 8$

$a_4 = a_3 + a_2 + a_1 + a_0 + 1 = 8 + 4 + 2 + 1 + 1 = 16$

It appears that $a_n = 2^n$ for $n \geq 0$. Prove this by strong induction:

Let P_n be the statement that $a_n = 2^n$ for $n \geq 0$.

Base Case:

$a_0 = 1 = 2^0$, therefore P_0 is true.

Inductive Step:

Show: $(P_0 \& P_1 \& \cdots \& P_{n-1}) \Rightarrow P_n$.

$a_n = a_{n-1} + a_{n-2} + \cdots + a_0 + 1$ By the definition of a_n.

$= 2^{n-1} + 2^{n-2} + \cdots + 2^0 + 1$ Since $P_0 \& P_1 \& \cdots \& P_{n-1}$ are true.

$= \frac{2^n - 2^0}{2-1} + 1$ By the lecture on closed forms.

$= 2^n - 1 + 1$

$= 2^n - 1 + 1$

Therefore P_n is true.