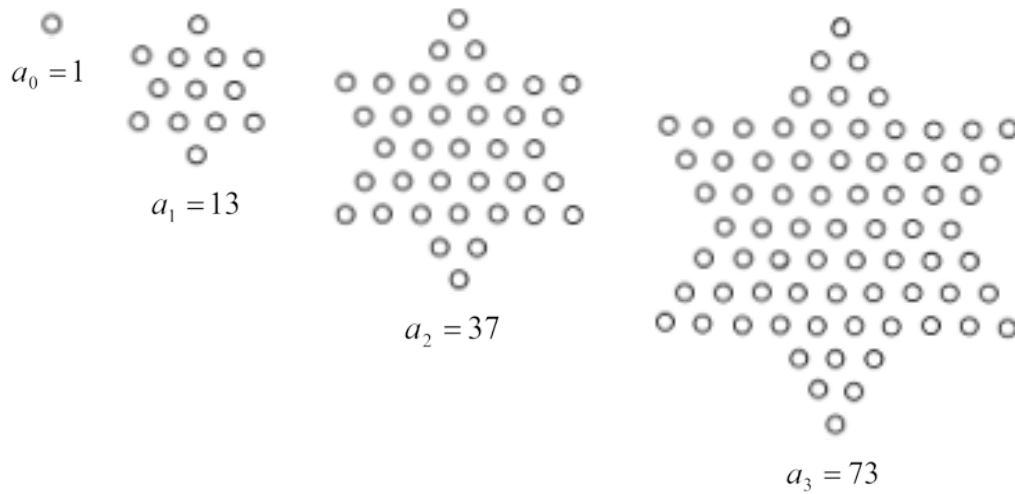


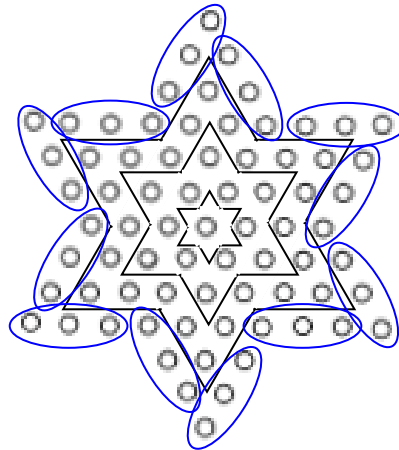
## Recursion and Induction: Solution

The game of Chinese Checkers is played on a board shaped like a six-sided star. For a star with side length  $n$ , we can consider how many holes there will be; the first few are done for you.



1. a) Write down a recurrence relation for the number holes in a star with side length  $n$ . You do not have to solve the recurrence relation.

**Solution:**



So the recurrence relation is:

$$\begin{aligned}
 a_0 &= 1 \\
 a_n &= a_{n-1} + 12n
 \end{aligned}$$

Extra:

We can solve this recurrence by iteration:

$$\begin{aligned}a_0 &= 1 \\a_1 &= 1 + 12 \cdot 1 \\a_2 &= 1 + 12 \cdot 1 + 12 \cdot 2 \\&\vdots \\a_n &= 1 + 12(1 + 2 + \dots + n) \\a_n &= 1 + 12 \frac{n(n+1)}{2} \\a_n &= 1 + 6n(n+1)\end{aligned}$$

b) Show by induction that number holes in a star with side length  $n$  is:

$$1 + 6n(n + 1)$$

Show by induction that:  $a_n = 1 + 6n(n + 1)$

Base Case

Let  $n = 0$

$$\begin{aligned}a_0 &= 1 \\1 + 0 \cdot (0 + 1) &= 1\end{aligned}$$



Inductive Step

Show:  $a_n = 1 + 6n(n + 1)$  (\*)  
 $\Rightarrow a_{n+1} = 1 + 6(n + 1)(n + 2)$

$$\begin{aligned}a_{n+1} &= a_n + 12(n + 1) && \text{by part a)} \\&= 1 + 6n(n + 1) + 12(n + 1) && \text{by (*)} \\&= 1 + 6(n + 1)(n + 2)\end{aligned}$$

