

NAME: _____

ID: _____

Power Plant: Solution

A power plant has an onsite telephone system with numbers that run from 000 to 999. Recently the plant's automated switchboard has been transposing two adjacent digits of the number dialed. Unfortunately the boss has decided to fire everyone that has a non-working site number, and this power plant employs exactly 501 workers. This power plant has put you in charge of devising a scheme where a switchboard error (transposing two adjacent digits) will only result in a call to an inoperative number. To save the jobs of plant workers devise a scheme where more than 500 telephones numbers are valid and the switchboard error can be detected. Note: a fourth digit cannot be added to the telephone numbers, such a solution would be too expensive for the power plant.

Solution.

There are 10^3 eligible numbers. Ten of them, where all 3 digits are identical, are called privileged numbers, since transposition has no effect on them. The other numbers are called ordinary numbers. In an ordinary number, there exist two distinct digits i and j with i to the left of j but not necessarily immediately to the left. The pair (i, j) is called an up-pair if $i < j$. Count all the up-pairs in an ordinary number. If the total number of up-pairs is even, call the number red. If the total number of up-pairs is odd, call the number blue. Note that when an ordinary number is misdialled by transposition, a red number becomes a blue number and a blue number becomes a red number. It follows that we can use all ten privileged number plus at least half of the ordinary numbers (the red ones or the blue ones, whichever have more). It follows that we can have more than $5 \cdot 10^2$ telephone numbers.