

Feedback #6 Coding Theory

Consider the following code:

```

0 0 0 0 0 0 0
0 0 0 1 0 1 1
0 0 1 0 1 0 1
0 0 1 1 1 1 0
0 1 0 0 1 1 0
0 1 0 1 1 0 1
0 1 1 0 0 1 1
0 1 1 1 0 0 0
1 0 0 0 1 1 1
1 0 0 1 1 0 0
1 0 1 0 0 1 0
1 0 1 1 0 0 1
1 1 0 0 0 0 1
1 1 0 1 0 1 0
1 1 1 0 1 0 0
1 1 1 1 1 1 1

```

Fact: The Hamming distance between any two codewords in the above code is 3 or more:

$$\min H(x, y) \geq 3$$

a) Show:

$$\min H(x, y) \leq 3$$

b) The fact together with part a) shows that: $\min H(x, y) = 3$. How many corrupted digits can the code detect?

c) How many corrupted digits can the code correct?

Solution.

a) Looking at the first two codewords in code:

```

x: 0 0 0 0 0 0 0
y: 0 0 0 1 0 1 1

```

We get $H(x, y) = 3$ and therefore: $\min H(x, y) \leq 3$.

b) $n + 1 = \min H(x, y) = 3 \Rightarrow n = 2$

c) $2n + 1 = \min H(x, y) = 3 \Rightarrow n = 1$