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Feedback #3 Modular Arithmetic

1. Find the number r where $0 \leq r < 11$ in each of the following:

- a) $43 \equiv r \pmod{11}$
- b) $414 \equiv r \pmod{11}$
- c) $-12 \equiv r \pmod{11}$
- d) $-137 \equiv r \pmod{11}$
- e) $689 \equiv r \pmod{11}$

2. Solve for x :

- a) $y \equiv 5x + 25 \pmod{26}$ (Answer: $x \equiv -5y - 5$)
- b) $y \equiv 11x + 10 \pmod{26}$ (Answer: $x \equiv 19y - 8$)
- c) $y \equiv 17x - 7 \pmod{26}$
- d) $y \equiv 25x + 4 \pmod{26}$
- e) $y \equiv 9x + 11 \pmod{26}$

Solution:

1.

- a) $43 \equiv 10 \pmod{11}$
- b) $414 \equiv 7 \pmod{11}$
- c) $-12 \equiv 10 \pmod{11}$
- d) $-137 \equiv 6 \pmod{11}$
- e) $689 \equiv 7 \pmod{11}$

1. Solve for x :

$$\begin{aligned} \text{a) } y &\equiv 5x + 25 \pmod{26} \\ &\Rightarrow -5y \equiv -25x + 5 \pmod{26} \\ &\Rightarrow -5y \equiv x + 5 \pmod{26} \\ &\Rightarrow x \equiv -5y - 5 \pmod{26} \end{aligned}$$

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b) $y \equiv 11x + 10 \pmod{26}$

$$\Rightarrow -7y \equiv -77x - 70 \pmod{26}$$

$$\Rightarrow -7y \equiv x + 8 \pmod{26}$$

$$\Rightarrow x \equiv 19y - 8 \pmod{26}$$

c) $y \equiv 17x - 7 \pmod{26}$

$$\Rightarrow -3y \equiv -3(-9)x + 21 \pmod{26}$$

$$\Rightarrow -3y \equiv x - 5 \pmod{26}$$

$$\Rightarrow x \equiv 23y + 5 \pmod{26}$$

d) $y \equiv 25x + 4 \pmod{26}$?

$$\Rightarrow -y \equiv -1(-1)x - 4 \pmod{26}$$

$$\Rightarrow -y \equiv x - 4 \pmod{26}$$

$$\Rightarrow x \equiv -y + 4 \pmod{26}$$

e) $y \equiv 9x + 11 \pmod{26}$?

$$\Rightarrow 3y \equiv 3(9)x + 33 \pmod{26}$$

$$\Rightarrow 3y \equiv x + 7 \pmod{26}$$

$$\Rightarrow x \equiv 3y - 7 \pmod{26}$$