

Feedback #2 Coins

There are two counterfeits among nine identically looking coins. The seven genuine coins all weigh the same. The two counterfeits weigh the same but are slightly heavier than the genuine coins. You have a **three-pan** balance at your disposal, but no other weights except for the coins themselves. You also have the ability to duplicate a single coin once. A genuine coin or a counterfeit coin can be duplicated; but the duplication process does not tell you if the coin is counterfeit or not. Identify the two counterfeit coins with the smallest number of weighings using the three-pan balance.

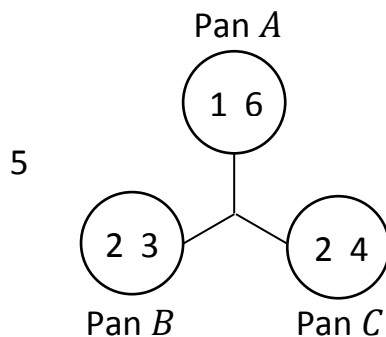
In a three-pan balance, the heaviest pan is lowest, the lightest pan is highest, and pans of the same weight are at the same level.

Solution: The problem cannot be solved in one weighing since there are “9 choose 2” = $\binom{9}{2} = 36$ different choices for the counterfeit coins but only 13 ways the three-pan balance can respond with one weighing. The problem can be solved in two weighings. Put two of the coins in each of the three pans and leave the rest aside.

Case 1: all pans balance. Then the counterfeits are not on the scale. This means there are two counterfeits among the three coins left aside. In the second weighing, place each of the three remaining coins in question on a different pan. Now, the two heaviest pans contain counterfeits.

Case 2: one pan is light. This means there are two counterfeits among the four coins on the heaviest pans. Of these four coins, put one in each pan and leave the final one aside for the second weighing. Now, the two heaviest pans contain counterfeits or the single heaviest pan contains a counterfeit and the final coin left aside is counterfeit.

Case 3: one pan is heavy. Call the coins on the heavy pan coins 1 and 2. Call the three coins left aside 3, 4, and 5. Pick one of the four coins on the two lightest pans and call it coin 6. In this case there is at least one counterfeit among coins 1 and 2 and there is at most one counterfeit among coins 3, 4, 5. Coin 6 is a genuine coin. In the second weighing duplicate coin 2 and use the following placement:



Order of pans (Lighter values are smaller)	Counterfeits
$A = B = C$	1 and 2
$C < B = A$	1 and 3
$B < C = A$	1 and 4
$B = C < A$	1 and 5
$A < C < B$	2 and 3
$A < B < C$	2 and 4
$A < B = C$	2 and 5

Therefore the smallest number of weighings needed is: 2