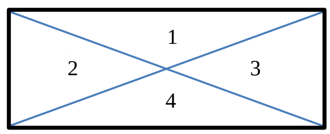
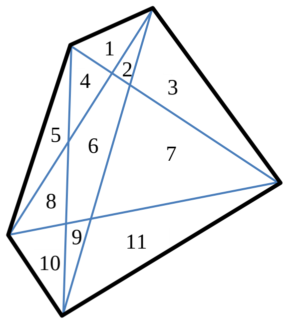
HINT Assignment 4 #1

- The theorem says that the problem can be solved by counting the number of diagonals and the number of interior intersection points.
- <u>Definition:</u> A region is called **convex** if for every pair of points within the region, every point on the straight line segment that joins them is also within the region.
- To help understand the question verify $a_4 = 4$ and $a_5 = 11$:





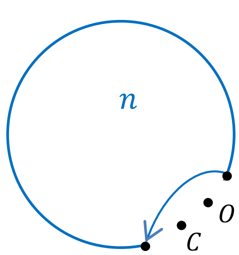
HINT Assignment 4 #4

Part of the inductive step is given.

<u>Inductive Step:</u>

Show: if it possible to find a starting position to travel around the city with n open and n closed gas stations then it possible to find a starting position to travel around the city with $n+1$ open and $n+1$ closed gas stations.

Search for an open gas station O which is followed by a closed gas station C. Notice that during travel around the city if one adds a gallon at O one can always travel one gas station past C. Noting that reduces the problem to finding a starting position among n open and n closed gas stations:



HINT Assignment 4 #5

The proof using strong induction is given below but the base cases are missing.

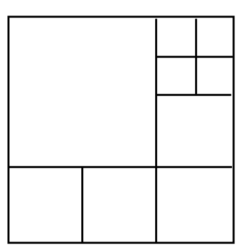
Let P_n be the statement for $n \geq 6$.

Base Cases:

Inductive Step:

Show: $P_n \Rightarrow P_{n+3}$.

Notice that if a square can be dissected into n smaller squares, then it can be dissected into $n + 3$ smaller squares. This is done by taking one of the existing squares and dissecting it into four squares of equal size. For example, using the dissection of 6 squares, the following is a dissection into 9 squares:



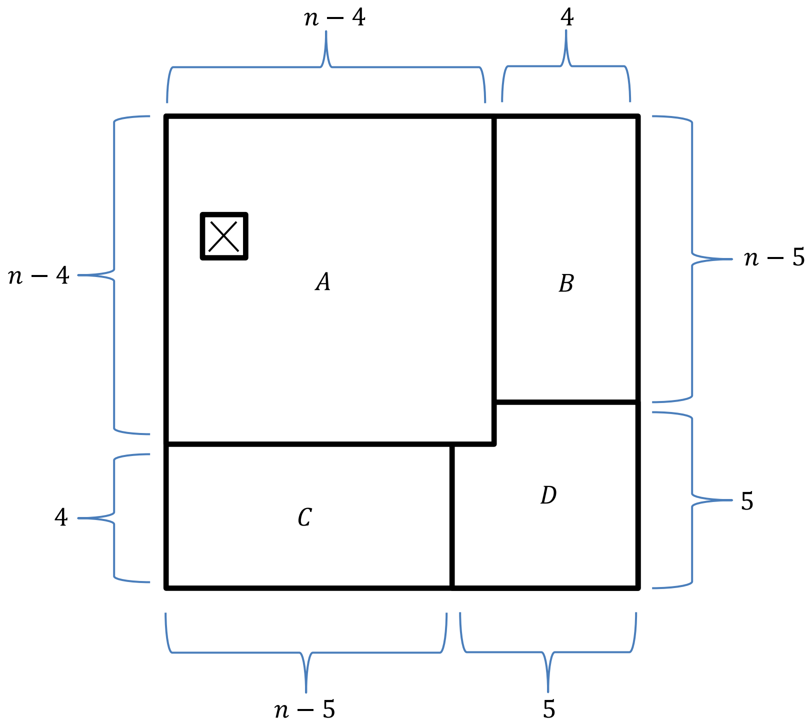
Therefore if P_n is true then P_{n+3} is also true.

HINT Assignment 4 #8b

Induction is **not** needed for this problem.

Note that the special cases of $n = 2, 8$ have both been shown in example 1 of lecture 16, so we can safely assume that $n \geq 11$.

Let $n \equiv 2 \pmod 3$ for $n \geq 11$ and section off a deficient $n \times n$ board as follows:



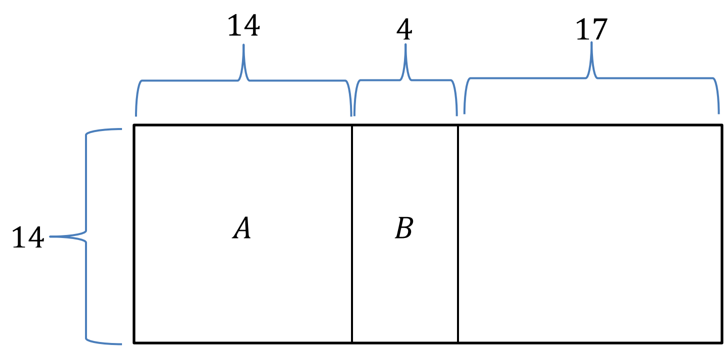
By symmetry when $n \geq 11$ the missing the missing square is in section A.

HINT Assignment 4 #9

Use the result from #8 to help solve this problem:

<u>Base case:</u> $n = 0$

Start by breaking the board into the following sections:



By symmetry we only need to consider the case when the missing square is in section A or B. In both cases use a 14×14 deficient square.