

# MATH 222

## Assignment#3

Due: 5pm on the date stated in the course outline.  
Hand in to the assignment box on the 3<sup>rd</sup> floor of CAB.

1. Using the keystream

|        |       |       |       |       |       |       |       |
|--------|-------|-------|-------|-------|-------|-------|-------|
|        | 11111 | 00000 | 11111 | 00000 | 11111 | 00000 | 11111 |
| decode | 11000 | 00000 | 10100 | 00101 | 01001 | 00000 | 00111 |

2. Decipher the following message which was encrypted using the seed I.

J S R V J

3. Decipher the following three messages:

a) Xli wigrh qiwweki aew irgvctxih ywmrk xli pmriev gshi amxl e xlvii erh o wmb.

b) Yw lw lbs vwoflb vnwwf wv MGJ, wt lbs jonnslet-jwgfz tsgfsil lbs snsglwf ei g ismfsl uwfp. Ois el lw psmezbsf lbs lbefp qsiigys.

c) Jzwck nhf! Bwm jrzx idzfgu jnpo ustbw.

4. The following message was encrypted using a linear code. Decipher the message.

“B ibatn ubag rq lbgwnlbgzhd pwzwh enhrlnd jdnqji knmnirunk pzgw bedrijgniv or kndzan gr en jdnqji, bok zo b dzgjbzro pwnan oerkv hrjik urddzeiv forp zo pwbg banb zg prjik enhrln jdnqji; bok gwnan pnan or tnonabi zokzhhgzrod gwbg zg nmna prjik en dr.”

-Crwo mro Onjlboo

5. Find a closed form for

$$S_3 = 1^3 + 2^3 + 3^3 + \dots + n^3$$

To find the closed form, use the expansion:

$$(x + 1)^4 = x^4 + 4x^3 + 6x^2 + 4x + 1$$

by plugging in  $x = 1, x = 2, x = 3, \dots, x = n$ .

6. Find a closed form for

$$a_n = 1^2 \cdot n + 2^2 \cdot (n - 1) + 3^2 \cdot (n - 2) + \dots + n^2 \cdot 1$$

for  $n \geq 1$ .

7. Let  $a_n$  be the number of words (strings) of length  $n$  that can be made using the digits  $\{0,1,2,3\}$  with an odd number of twos. Find a recurrence relation for  $a_n$  and solve the recurrence. The first few values of  $a_n$  are calculated for you.

$$a_1 = 1$$

2

$$a_2 = 6$$

20 21 23 02 12 32

$$a_3 = 28$$

200 201 203 210 211 213 230 231 233 222  
 020 021 023 002 012 032  
 120 121 123 102 112 132  
 320 321 323 302 312 332

8. Let  $a_n$  be the number of ways can you tile a  $2 \times n$  rectangle using dominoes and square tetrominoes. Dominoes are of size  $2 \times 1$  and tetrominoes are of size  $2 \times 2$ . Find a recurrence relation for  $a_n$ ; you do not have to solve this recurrence. The first few values of  $a_n$  are calculated for you.

$$a_1 = 1$$



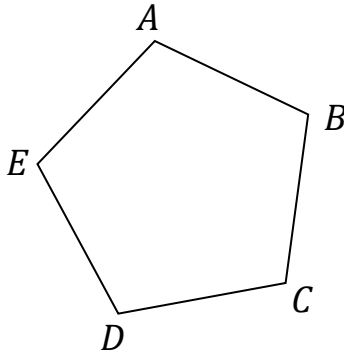
$$a_2 = 3$$



$$a_3 = 5$$



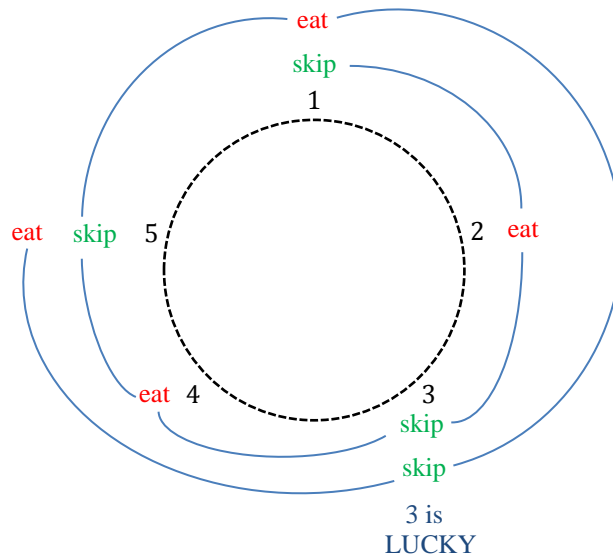
9. Let  $a_n$  be the number of ways a bug can start at vertex  $A$  of the regular pentagon below and reach vertex  $C$  for the first time after  $n$  moves. A move is made from one vertex to an adjacent vertex. Find a recurrence relation for  $a_n$ ; you do not have to solve this recurrence. The first four values of  $a_n$  are:  $a_1 = 0, a_2 = 1, a_3 = 1, a_4 = 2$ .



10. Sylvester caught  $n$  mice which he arranged in a circle and numbered them  $1, 2, \dots, n$  in clockwise order. Starting with mouse number 1, Sylvester went around the circle in clockwise order, skipping over one mouse and eating the next one. He went round and round by the same rule, until only one mouse was left. This lucky mouse was then set free. Denote  $f(n)$  as the number assigned to the lucky mouse initially. Now

$$f(1) = 1, f(2) = 1, f(3) = 3, f(4) = 1, \text{ and } f(5) = 3.$$

For example to find  $f(5)$ :



(a) Express  $f(2n)$  and  $f(2n + 1)$  in terms of  $f(n)$

(b) Given that  $f(530) = 37$ , determine  $f(2121)$ .

*Bonus.*



The Mayor of Edmonton wants Dr. Ecco to help him with plans for the new arena. Dr. Ecco isn't interested so the Mayor sends his goons to bring him in. Dr. Ecco sends out a decoy cipher text to mislead the goons. The cipher text is as follows:

O oy pwknmxg m pyrfe ob ZQNP

Except for the last word, the mayor has found the plain text:

I am hosting a party in

- a) Dr. Ecco's decoy cipher text was encoded using the Hill cipher with the encoding function:

$$E(x_{even}) \equiv 5 \cdot x_{even} + 6 \cdot x_{odd} \pmod{26}$$

$$E(x_{odd}) \equiv 18 \cdot x_{even} + x_{odd} \pmod{26}$$

Complete the decoy message by finding the decoding function and decode the last word of the cipher text. If the mayor's goons follow Dr. Ecco's decoy message where will they end up going?

- b) Dr. Ecco has double-encoded the last word in his message. To get the real message, you must suppose that the plain text word found in part a) is also cipher text. What is the real message?