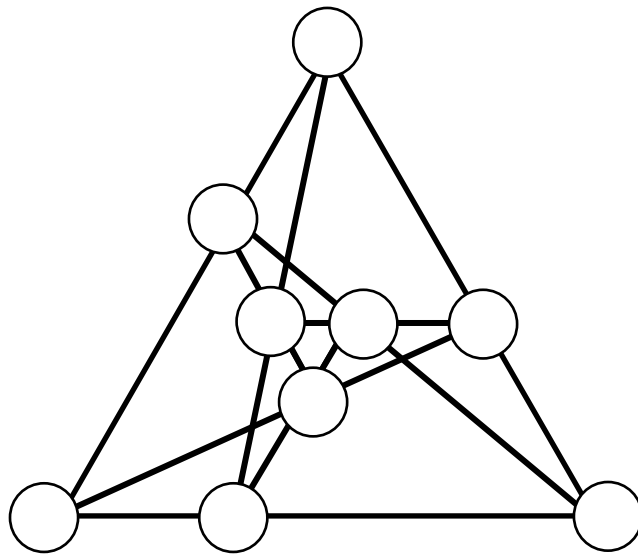


MATH 222

Assignment#2

Due: 5pm on the date stated in the course outline.
Hand in to the assignment box on the 3rd floor of CAB.

1. The following game is called Tri-Tic-Tack-Toe. How to play: Player X and Player O alternately put their symbol in an empty circle on their turn. Whoever makes three in a row on any straight line wins the game. Each player can only take four turns.



State whether Tri-Tic-Tack-Toe ends in a theoretical draw or if there is a winning strategy. If there is a winning strategy, which player can always win: the first player or the second player? If the first player can always win, state the first move that he or she needs to make. If the second player can always win, state the first move he or she must make in response the 9 ways the game may be begin.

2. Let's play Nim. This version of the game will have 5 rows. The rows will have the following amount of counters:

Row 1: 30
Row 2: 31
Row 3: 42
Row 4: 76
Row 5: 104

- a) Explain why the 1st player can always win this game.
- b) Indicate all of the opening moves that can guarantee victory for the first player.

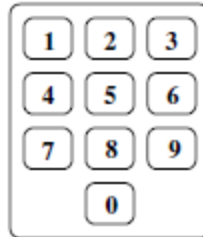
3. There were 5 children in the Emergency Room and between them they had stuck a total of 76 beans up their noses. Show that there must be three children with a combined total of 46 or more beans up their noses.
4. Prove that there exists two powers of two which differ by a multiple of 2020.
5. In Las Vegas on a particular weekend night there are 101 whales (a nickname given to high stakes gamblers) who randomly pick four different casinos to gamble at.
 - a) Prove that there is a pair of casinos that will host a combined total of at least 51 whales.
 - b) Out of the four casinos we can make a total of 6 pairs of casinos. How many pairs out of the 6 pairs will host a total of at least 51 whales? Your answer **must** include one of the quantifiers “at least”, “exactly” or “at most”.
6. A counselor has $3n + 2$ campers with her at a junction in a hiking trail. She knows their camp is twenty minutes down one of four possible paths. It will be dark in one hour and the group must find their camp before dark. n of the $3n + 2$ campers sometimes lie, and unfortunately the counselor doesn't know which n they are.
 - a) Is there a way for the counselor to accurately deduce the location of the camp?
 - b) Prove this problem cannot be solved with only $3n + 1$ campers.
7. A different subset of 32 natural numbers less than 64 is printed on each of the 6 cards below according to the following rules:

Card 1: The 1st digit in the binary expansion of each number on this card is a one.
Card 2: The 2nd digit in the binary expansion of each number on this card is a one.
Card 3: The 3rd digit in the binary expansion of each number on this card is a one.
Card 4: The 4th digit in the binary expansion of each number on this card is a one.
Card 5: The 5th digit in the binary expansion of each number on this card is a one.
Card 6: The 6th digit in the binary expansion of each number on this card is a one.

Note: the 1st (rightmost) digit in the binary expansion appears in the rightmost position while the 6th digit in the binary expansion appears in the leftmost position.

I am looking at a number that appears on cards 1, 3, and 6 but not on cards 2, 4, and 5. What number am I looking at?

8. The telephone numbers in a town run from 00000 to 99999. There are a number of errors that commonly occur:
- The Diagonal Error: a common error in dialling on a standard keypad is to punch in a digit diagonally adjacent to the intended one. So on a standard dialling keypad, 4 could erroneously be entered as 2 or 8 (but not as 1, 5, or 7).



- The Switch Bug: After a correct number has been called, in transmission, one pair of adjacent digits gets swapped. For example the number *abcde* could be called but whoever is at *abcd* receives the phone call.
- The Smudged Digit: A single digit gets corrupted in transmission and it is known which digit got corrupted. For example: 38754 is dialed but in transmission, it becomes 387□4.
- The Digit Error: A single digit gets changed into another random digit during transmission and it is unknown which digit was changed. For example 38754 is dialed but 38757 receives the call.

It has been decided that a sixth digit will be added to each phone number. Given a phone number *abcde*, there are three different proposed schemes:

Code 1: $a + b + c + d + e + X \equiv 0 \pmod{10}$

Code 2: $2a + b + 2c + d + 2e + X \equiv 0 \pmod{10}$

Code 3: $6a + 5b + 4c + 3d + 2e + X \equiv 0 \pmod{10}$

Fill out the following chart with a Yes or a No (you DO NOT have to show your work).

	Can detect a Diagonal Error?	Can detect a Switch Bug?	Can correct a Smudged Digit?	Can detect a Digit Error?
Code 1				
Code 2				
Code 3				

9. The following message is received using the 15-digit Hamming Code. Correct any single error that may have occurred during transmission.

<i>a</i>	<i>a</i>	<i>a</i>	<i>a</i>		<i>a</i>	<i>a</i>	<i>a</i>	<i>a</i>						
<i>b</i>	<i>b</i>	<i>b</i>		<i>b</i>	<i>b</i>			<i>b</i>	<i>b</i>					
<i>c</i>	<i>c</i>		<i>c</i>	<i>c</i>		<i>c</i>		<i>c</i>	<i>c</i>	<i>c</i>				
<i>d</i>		<i>d</i>	<i>d</i>	<i>d</i>		<i>d</i>	<i>d</i>	<i>d</i>	<i>d</i>	<i>d</i>	<i>d</i>			<i>d</i>
0	1	0	1	1	0	1	0	0	1	1	1	0		

10. I am thinking of a number between 0 and 15. You ask me to write the digit in binary form (using 4 digits) and ask the following seven questions.

Is the 1st (rightmost) digit in the binary expansion of your number a one?

Is the 2nd digit in the binary expansion of your number a one?

Is the 3rd digit in the binary expansion of your number a one?

Is the 4th digit in the binary expansion of your number a one?

Is there an odd number of ones in the 2nd, 3rd, and 4th positions?

Is there an odd number of ones in the 1st, 3rd, and 4th positions?

Is there an odd number of ones in the 1st, 2nd, and 4th positions?

My answers to your questions in the order you asked them are:

No, Yes, Yes, No, Yes, No, No.

However, I may have lied once. What is my number?

Bonus question.

A different subset of the natural numbers less than 64 is printed on each of the 6 cards below according to the following rules:

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Card 2: The 2nd digit in the binary expansion of each number on this card is a one.

Card 3: The 3rd digit in the binary expansion of each number on this card is a one.

Card 4: The 4th digit in the binary expansion of each number on this card is a one.

Card 5: The 5th digit in the binary expansion of each number on this card is a one.

Card 6: The 6th digit in the binary expansion of each number on this card is a one.



Dr. Ecco found a twin prime number that appears on cards 1, 3, and 6 but not on cards 2, 4, and 5. Find Dr. Ecco's number.