MATH 222

Assignment#1

Due: 5pm on the date stated in the course outline.
Hand in to the assignment box on the 3rd floor of CAB.

1. Four people are being pursued by a menacing beast. It is nighttime, and they need to cross a bridge to reach safety. It is pitch black, and only two can cross at once. They need to carry a lamp to light their way. The first person can cross the bridge in no less than 10 minutes, the second in 5 minutes, the third in 2 minutes, and the fourth in 1 minute. If two cross together, the couple is only as fast as the slowest person. (That is, a fast person can't carry a slower person to save time, for example. If the 10-minute person and the 1-minute person cross the bridge together, it will take them 10 minutes.) The person or couple crossing the bridge needs the lamp for the entire crossing and the lamp must be carried back and forth across the bridge (no throwing, etc.) If they don't all get completely across in less than 18 ½ minutes, whoever is on the bridge or left behind will be eaten by the beast. Is it possible for all of them to get across?

2. Find the multiplicative inverse of 13 in mod 30.

3. There is only one way to open the safe below.

4D	4D	1L	3L	Open
2R	1D	1U	2L	4L
4R	1L	2D	1U	2L
4R	2R	2L	1D	2U
4R	1U	1U	4U	4U

   You must press each button exactly once in the correct order in order to reach Open. Each button is marked with a direction: U is up, L is left, R is right, and D is down. The number of spaces to move is also marked on each button. Which button is the first one you must press?

4. There are 9 coins, all identical except that one is counterfeit and is a heavier than the others. Show how to find the counterfeit in two weighings using a pan balance.

5. There are 10 coins, all identical except that one is counterfeit and is a different weight than the others. It is not known whether the counterfeit is heavier or lighter. Show how to find the counterfeit in three weighings using a pan balance.

6. Determine if each of the following pairs of integers are congruent modulo 11.

   a) 2 , 218
   b) 0 , 242
   c) 4 , 420
   d) -7 , 7
   e) -31 , 68
   f) -1 , 120

7. In 2020, Valentine's Day is on a Friday. What day will Valentine's Day be in the year 3000? (Don't forget that every year divisible by 4 is a leap year unless it is divisibly by 100. That is, 2100, 2200, 2300, etc are not leap years. An exception is that centuries are leap years when they are divisible by 400; so 2000 was a leap year and so will be 2400). Hint: There will be 238 occurrences of February 29th between February 14th 2020 and February 14th 3000.

8. Show that none of the integers:

   $$11, 111, 1111, 11111, \ldots$$

   is the square of another integer.
   Hint: Pick any number from the above list:

   $$111 \cdots 11$$

   Now note that $111 \cdots 11 \equiv 111 \cdots 100 + 11 \equiv 0 + 3 \equiv 3 \pmod 4$.

9. Five darts are thrown at a square target measuring 14 inches on a side. Prove that two of them must be at a distance no more than 10 inches apart.

10. There are n coins of random integer values coin lined up in a row on a table. Nora picks a coin from one of the ends of the row. Next, Abraham picks up a coin from one of the ends of the remaining row of coins. They alternate in this manner until the last coin is picked up. Nora wins the game if she has picked up at least the same amount of money as Abraham.

    a) Find all the positive integer values for n (the number of coins) where Nora can always win regardless of the values of each coin. Provide a reason why.

    b) Find all the positive integer values for n where Nora cannot always win regardless of the values of each coin. To show that Nora cannot always win provide a counterexample with specific values for each of the n coins where Abraham can always win.

Bonus question

Suddenly a stern knock on Dr. Ecco's door, and in walked Michael Monetary. The man worked inside the government and was in charge of a coin factory. He stated his problem: "one of my coin making machines is not working properly. It produces coins in batches of 15; exactly one out of every batch has an incorrect weight. The first coin from a batch is always perfect, the second if it is the one of incorrect weight is never too light, and otherwise the bad coin could end up being heavier or lighter than it's supposed to be. I would like my workers to quickly find and remove the bad coin from each batch using a regular pan balance in 3 weighings. Anymore weighings and I will surely lose my job over production losses. Finally, I would like to keep track of the bad coins being too heavy or to light; this statistic could help fix the machine."

After a few minutes Ecco handed Mr. Monetary a scrap of paper and said "here is your solution in three weighings, it's a good thing the first coin from each batch is always perfect."

How did Ecco solve Mr. Monetary's problem?