PART 1: Multiple Choice

1) Consider the 7-bit Hamming Code. Which of the following statements are true?

I. A maximum of 1 corrupted digit can be corrected in a codeword.

II. A maximum of 2 corrupted digits can be detected in a codeword.

III. The code can correct errors but cannot detect errors.

Min Hamming distance ?

A. I only.

B. II only.

C. III only.

D. I and II only.

E. I and III only.

- The Hamming Code can correct 1 corrupted digit,

$\therefore$ min $H(x,y) \geq 3$

- There is an example where $H(x,y)=3$:

a	a	a		a		
b	b		b		b	
c		c	c			c
0	0	0	0	0	0	0

a	a	a		a		
b	b		b		b	
c		c	c			c
0	0	0	1	0	1	1

$\therefore$ min $H(x,y) \leq 3$

2) There are 50 coins, all identical except that one is counterfeit and is a different weight than the others. It is not known whether the counterfeit is heavier or lighter. Dr. Ecco, using a pan balance, created a scheme that identifies the counterfeit with a minimum number of weighings. How many weighings does Dr. Ecco's scheme use?

A. 2

B. 3

C. 4

D. 5

E. 6

weighings	At Most How Many Coins ?
1	$\lceil 3^1/2 \rceil = 2$
2	$\lceil 3^2/2 \rceil = 5$
3	$\lceil 3^3/2 \rceil = 14$ (Can be done with a good coin A1)
4	$\lceil 3^4/2 \rceil = 41$
5	$\lceil 3^5/2 \rceil = 122$

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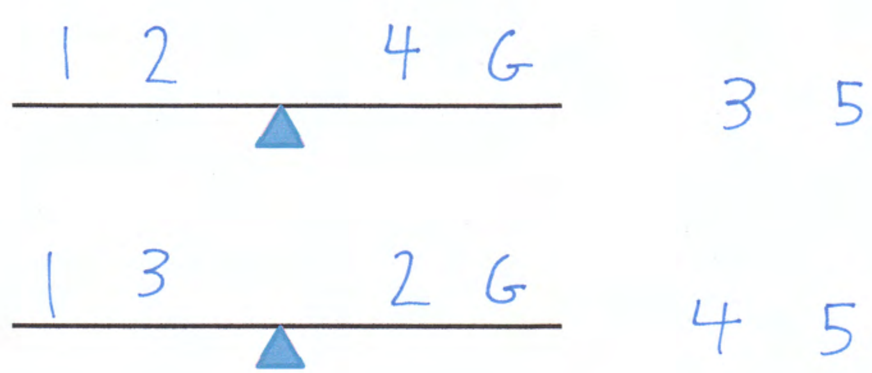
PART 2: Fill In the Blank

1. There are 5 coins, all identical except that one is counterfeit and is a different weight than the others. It is not known whether the counterfeit is heavier or lighter. In addition to these 5 coins you have at your disposal one more coin that is known not to be counterfeit. Show how to find the counterfeit in two weighings using a pan balance. A non-adaptive solution is required.

   Label the coin that is known not to be counterfeit *G* and label the other five coins 1,2,3,4,5.

Weigh the six coins in the following pattern:



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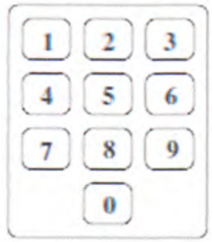
2. The telephone numbers in town run from 00000 to 99999; a common error in dialling on a standard keypad is to punch in a digit **horizontally** adjacent to the intended one. So on a standard dialling keypad, 4 could erroneously be entered as 5 (but not as 1, 2, 7, or 8). No other kinds of errors are made.



It has been decided that a sixth digit X will be added to each phone number $abcde$. There are three different proposals for the choice of X:

Code 1: $a + b + c + d + e + X \equiv 0 \pmod 2$

Code 2: $6a + 5b + 4c + 3d + 2e + X \equiv 0 \pmod 6$

Code 3: $6a + 5b + 4c + 3d + 2e + X \equiv 0 \pmod{10}$

Out of the 3 codes given, choose one that can detect a horizontal error and one that cannot detect a horizontal error.

Suppose a horizontal error changed $a \leftrightarrow a'$.

Code 1
- Case 1 $a' + b + c + d + e + X \not\equiv 0 \pmod 2$
  $\therefore$ report an error
- Case 2
$$\begin{aligned} & a' + b + c + d + e + X \equiv 0 \pmod 2 \\ - \; & a + b + c + d + e + X \equiv 0 \pmod 2 \\ \hline & a' - a \equiv 0 \pmod 2 \end{aligned}$$
$\therefore$ $a' - a$ is even & $a' - a = \pm 1$ ⚡

Code 2 Both 100000 & 200000 are valid code word

Code $\boxed{2}$ cannot detect a horizontal error.

Code $\boxed{1}$ can detect a horizontal error.

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Code 3

Let
- a horizontal error change $y \leftrightarrow y'$ where $y \in \{a, b, c, d, e, x\}$.
- "A" represent Code 3's equation after dialling.
- "B" ——————————— before ——.

CASE 1 If A is not true report an error.

CASE 2 If A is true find A − B:

$n(y' - y) \equiv 0$ $\qquad n \in \{1, 2, 3, 4, 5, 6\}$

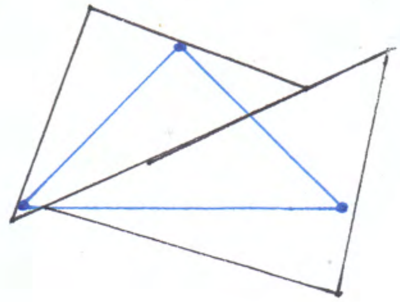
$\Longrightarrow \quad n \equiv 0\,(y' - y)^{-1}$ $\qquad$ Since $y' - y \equiv \pm 1$ has an inverse in (mod 10)

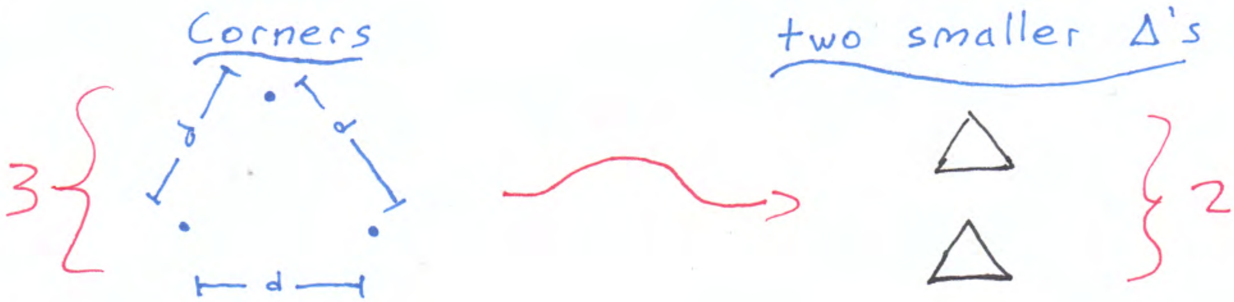
$\Longrightarrow \quad n \equiv 0 \pmod{10}$ & $\quad n = 1$ or 2 or 3 or 4 or 5 or 6

$↯$

3. Can an equilateral triangle be covered with two smaller equilateral triangles?

Suppose an equilateral triangle can be covered by two smaller equilateral triangles:





∴ one smaller triangle contains at least 2 corners.

∴ 2 corner have a distance apart of less than d.

Therefore an equilateral triangle *(circle one)*

CAN CANNOT

be covered with two smaller equilateral triangles.

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4. Consider the following list of binary numbers (it goes on forever):

$$\begin{array}{c} 1 \\ 101 \\ 10001 \\ 1000001 \\ 100000001 \\ 10000000001 \\ \vdots \end{array}$$

$$\begin{array}{c} 2^0 \\ 2^2 + 2^0 \\ 2^4 + 2^0 \\ \vdots \\ 2^{2n} + 1 \end{array}$$

Which of these integers is divisible by 5?

Find $2^{2n}+1 \pmod 5$:

$$\begin{array}{ccc} 2^0 \equiv 1 & , \quad 2^4 \equiv 1 & , \ldots \\ 2^2 \equiv 4 & \quad 2^6 \equiv 4 & \end{array}$$

$$\therefore \quad 2^{2n} \equiv \begin{cases} 1 & n \text{ even} \\ 4 & n \text{ odd} \end{cases}$$

$$\therefore \quad 2^{2n} + 1 \equiv \begin{cases} 2 & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$$

(Describe which binary numbers in the above binary tree are divisible by 5).

$\therefore$ every 2^{nd} binary # is divisible by 5

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