MATH 222

Midterm

March 2nd, 2012

Part 1: Multiple Choice

- each question is worth 10 marks
- circle the best answer

Part 2: Fill In the Blanks

- each question is worth 20 marks
- the correct answer is worth full marks
- you may show your work for part-marks

Name: _ SAMPLE _ ID: ___ 2 ___

PART 1: Multiple Choice

1) Consider the 7-bit Hamming Code. Which of the following statements are true?

   I. A maximum of 1 corrupted digit can be corrected in a codeword.

   II. A maximum of 2 corrupted digits can be detected in a codeword.

   III. The code can correct errors but cannot detect errors.

   A. I only.

   B. II only.

   C. III only.

   D. I and II only.

   E. I and III only.

2) There are 50 coins, all identical except that one is counterfeit and is a different weight than the others. It is not known whether the counterfeit is heavier or lighter. Dr. Ecco, using a pan balance, created a scheme that identifies the counterfeit with a minimum number of weighings. How many weighings does Dr. Ecco's scheme use?

   A. 2

   B. 3

   C. 4

   D. 5

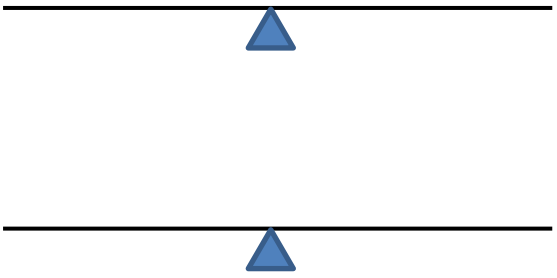
   E. 6

PART 2: Fill In the Blank

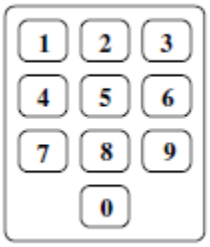
1. There are 5 coins, all identical except that one is counterfeit and is a different weight than the others. It is not known whether the counterfeit is heavier or lighter. In addition to these 5 coins you have at your disposal one more coin that is known not to be counterfeit. Show how to find the counterfeit in two weighings using a pan balance. A non-adaptive solution is required.

   Label the coin that is known not to be counterfeit G and label the other five coins $1,2,3,4,5$.

Weigh the six coins in the following pattern:



2. The telephone numbers in town run from 00000 to 99999; a common error in dialling on a standard keypad is to punch in a digit **horizontally** adjacent to the intended one. So on a standard dialling keypad, 4 could erroneously be entered as 5 (but not as 1, 2, 7, or 8). No other kinds of errors are made.



It has been decided that a sixth digit X will be added to each phone number $abcde$. There are three different proposals for the choice of X:

Code 1: $a + b + c + d + e + X \equiv 0 \pmod 2$

Code 2: $6a + 5b + 4c + 3d + 2e + X \equiv 0 \pmod 6$

Code 3: $6a + 5b + 4c + 3d + 2e + X \equiv 0 \pmod{10}$

Out of the 3 codes given, choose one that can detect a horizontal error and one that cannot detect a horizontal error.

Code ☐ cannot detect a horizontal error.

Code ☐ can detect a horizontal error.

3. Can an equilateral triangle be covered with two smaller equilateral triangles?

Therefore an equilateral triangle *(circle one)*

CAN CANNOT

be covered with two smaller equilateral triangles.

4. Consider the following list of binary numbers (it goes on forever):

1
1 0 1
1 0 0 0 1
1 0 0 0 0 0 1
1 0 0 0 0 0 0 0 1
1 0 0 0 0 0 0 0 0 0 1
⋮

Which of these integers is divisible by 5?

(Describe which binary numbers in the above binary tree are divisible by 5).