PART 1: Multiple Choice

1) Consider a binary code that has a minimum Hamming distance of 8 between each pair of codewords.

   A. A maximum of 6 corrupted digits can be detected in a codeword.

   B. 8 corrupted digits can be detected in a codeword.

   C. A maximum of 2 corrupted digits can be corrected in a codeword.

   D. 4 corrupted digits can be corrected in a codeword.

   E. A maximum of 7 corrupted digits can be detected in a codeword and a maximum of 3 corrupted digits can be corrected in a codeword.

Detect ?

$n+1 = 8$

$\Rightarrow n = 7$

Correct ?

$2n+1 = 8$

$\Rightarrow n = 7/2$

this means we can correct a maximum of $\lfloor 7/2 \rfloor = 3$ (round down)

2) There are 100 coins, all identical except that one is counterfeit and is heavier than the others. Dr. Ecco, using a pan balance, created a scheme that identifies the counterfeit coin with a minimum number of weighings. How many weighings does Dr. Ecco's scheme use?

   A. 2

   B. 3

   C. 4

   D. 5

   E. 6

weighings	MAX coins
1	3
2	$3^2 = 9$
3	$3^3 = 27$
4	$3^4 = 81$
5	$3^5 = 243$

PART 2: Fill In the Blank

1. A counselor and her campers are at a junction in a hiking trail and they know their campsite is 20 minutes down one of four paths. It will be dark in one hour so the counselor wants to send campers down all the paths to see which one leads to camp (the counselor can check a path too). They will then rendezvous in 40 minutes and choose which path to follow.

   What is the smallest number of campers the counselor will need if 10 of them sometimes lie?

A2's classification of the Camper's Problem gives:

$$3n + 2 = 3(10) + 2 = 32$$

as the smallest # of campers.

The smallest number of campers the counselor will need is $\boxed{32}$

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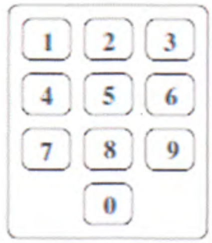
2. The telephone numbers in town run from 00000 to 99999: a common error in dialling on a standard keypad is to punch in a digit **vertically** adjacent to the intended one. So, on a standard dialling keypad, 4 could erroneously be entered as 1, or 7 (but not as 2, 5, or 8). No other kinds of errors are made.



It has been decided that a sixth digit X will be added to each phone number $abcde$. There are two different proposals for the choice of X:

Code 1: $a + b + c + d + e + X \equiv 0 \pmod{10}$

Code 2: $6a + 5b + 4c + 3d + 2e + X \equiv 0 \pmod{10}$

Only one of the two codes can detect the described error. Is it code 1 or code 2?

Code 1 can detect any single digit error.

(by A 2)

Note: Code 2 can not detect if a vertical error changes,

000000 into 080000

Code $\boxed{1}$ can detect this error.

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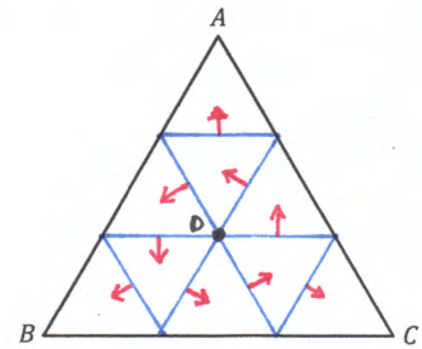
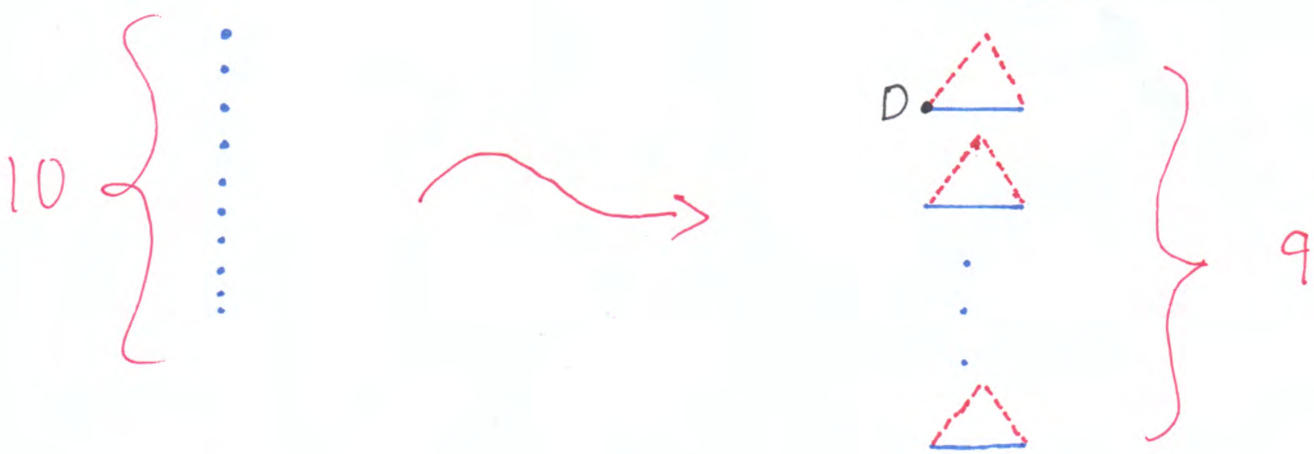
3. Let the triangle ABC be equilateral with $AB = 3$. If we select 10 points in the interior of this triangle is it true that there must be at least two whose distance apart is **strictly less than 1**?



Divide $\triangle ABC$ into 9 equilateral Δ's each with only one side and attach D to one Δ.

Points

Δ's with only one side and D



$\therefore$ one Δ will contain at least 2 points $<$ 1 unit apart.

Therefore out of 10 points in the interior of ΔABC there *(circle one)*
WILL   WILL NOT
be at least two whose distance apart is strictly less than 1.

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4. Consider the following list of binary numbers (it goes on forever):

$$\begin{array}{c} 1 \\ 1\,1 \\ 1\,0\,1 \\ 1\,0\,0\,1 \\ 1\,0\,0\,0\,1 \\ 1\,0\,0\,0\,0\,1 \\ \vdots \end{array} \qquad \begin{array}{c} 2^0 \\ 2^1 + 2^0 \\ 2^2 + 2^0 \\ \vdots \\ 2^n + 1 \end{array}$$

Which of these integers is divisible by 5?

Find $2^n + 1 \pmod 5$:

$$\begin{array}{l} 2^0 \equiv 1 \\ 2^1 \equiv 2 \\ 2^2 \equiv 4 \\ 2^3 \equiv 3 \end{array} \quad , \quad \begin{array}{l} 2^4 \equiv 1 \\ 2^5 \equiv 2 \\ 2^6 \equiv 4 \\ 2^7 \equiv 3 \end{array} \quad , \ldots$$

$$\therefore \quad 2^n \equiv \begin{cases} 1 & n \equiv 0 \pmod 4 \\ 2 & n \equiv 1 \pmod 4 \\ 4 & n \equiv 2 \pmod 4 \\ 3 & n \equiv 3 \pmod 4 \end{cases}$$

$$\therefore \quad 2^n + 1 \equiv \begin{cases} 2 & n \equiv 0 \pmod 4 \\ 3 & n \equiv 1 \pmod 4 \\ 0 & n \equiv 2 \pmod 4 \\ 4 & n \equiv 3 \pmod 4 \end{cases}$$

(Describe which binary numbers in the above binary tree are divisible by 5).

$\therefore$ every 4^{th} binary # starting at 101 is divisible by 5.

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