CMPUT 204 (B1) – Assignment # 4

DUE DATE: August 16, 2000

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1. *First Order Nonhomogeneous Linear Difference Equations*

   Consider the following 1st order nonhomogeneous linear difference equation

   $$\begin{aligned} a_n &= c_n a_{n-1} + d_n, \quad n \geq 1 \\ a_0 &= d_0 \end{aligned}$$

   where the coefficients c_n and d_n may both depend on n.

   (a) Prove by induction that a particular solution to this problem is given by

   $$a_n = \sum_{j=0}^{n} \left(\prod_{i=j+1}^{n} c_i \right) d_j$$

   for $n \geq 0$. Here we interpret the empty product as $\prod_{i=k}^{n} c_i = 1$ for $k > n$.

   (b) In the last assignment, we saw that the average number of comparisons done by the function `selection` in finding the k^{th} smallest entry in a list with n entries satisfied the discrete initial value problem

   $$\begin{aligned} a_n &= \frac{n a_0 + 2\left[a_1 + 2a_2 + 3a_3 + \cdots + (n-1)a_{n-1}\right]}{n^2} + n - 1, \quad n \geq 1 \\ a_0 &= 0. \end{aligned}$$

   Use the result in part (a) to find the exact solution to this discrete initial value problem.

2. *Unicyclic Graphs*

   If $G = (V, E)$ is an undirected graph with vertex set V and edge set E, and if $p = |V|$ is the number of vertices and $q = |E|$ is the number of edges, we say that G is *unicylic* if and only if G is connected and G contains exactly one cycle.

   Show that the following statements are equivalent, that is, either they are all true or they are all false.

   (a) G is unicyclic.

   (b) G is connected and $p = q$.

   (c) G has exactly one cycle and $p = q$.

3. *Connected Graphs and Spanning Trees*

   Let $G = (V, E)$ be an undirected graph with $p = |V|$ vertices and $q = |E|$ edges.

   (a) Show that G is connected if and only if G has a spanning tree.

   (b) If G is connected, what is the smallest number γ of edges that must be removed from G in order that no cycles remain? Express γ in terms of p and q.

   Note: The number γ is called the **cycle rank** or the **cyclomatic number** of the graph G.

   (c) Show that if

   $$q > \frac{(p-1)(p-2)}{2},$$

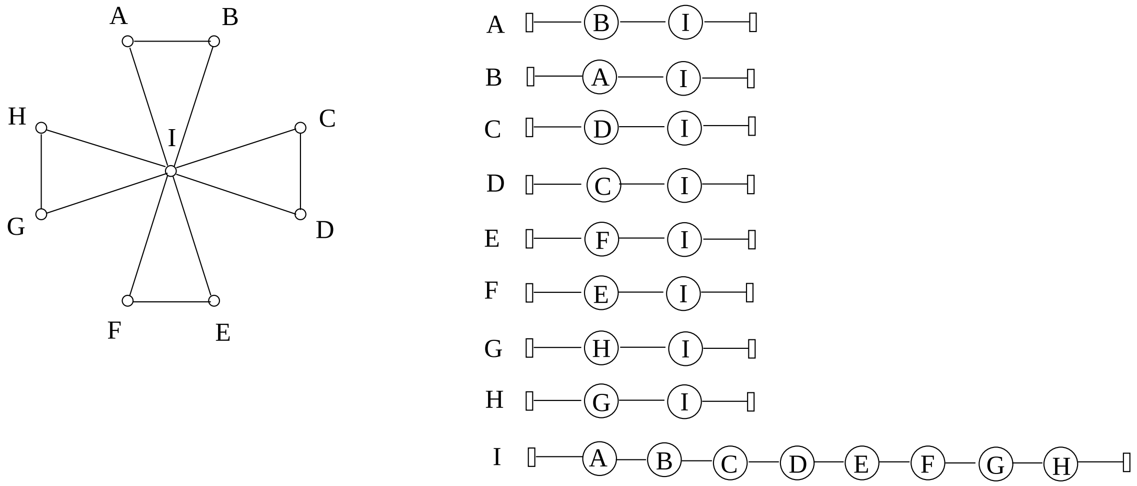
   then G is connected.

   (d) If G is connected, show that the height of any DFS spanning tree for G starting from a given root v must be at least as large as the height of a BFS spanning tree for G with root at v.

   (e) If G is connected, by starting with different roots, it might be possible for a DFS spanning tree for G to have a smaller height than a BFS spanning tree for G. Find a graph G in which this can happen.

4. *Depth First Search and Breadth First Search Trees*

Consider the graph G below with the adjacency lists shown.

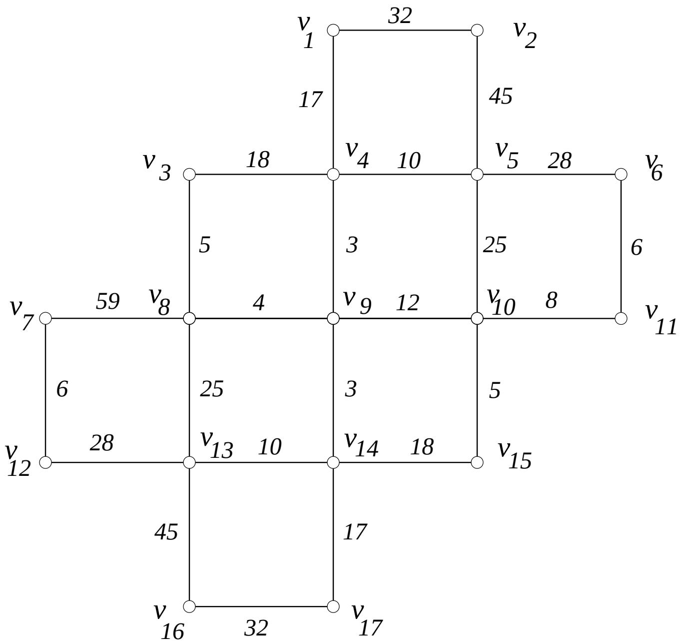


(a) Draw the DFS tree for this graph when the depth first search algorithm is executed with this adjacency list structure as input. Show the back edges also as dotted lines.

(b) Draw the BFS tree for this graph when the breadth first search algorithm is executed with this adjacency list structure. Also, show the contents of the queue for the queue-based implementation of the algorithm.

5. *Kruskal's Algorithm for Minimal Spanning Trees*

Use Kruskal's algorithm to find a minimal spanning tree for the weighted graph shown below.



What is the weight $W(T)$ of a minimal spanning tree?