Math 334—Assignment 4

1. (10 pts) Use the convolution theorem to obtain a formula for the solution to the initial value problem

$$y'' + y = g$$
, $y(0) = 0$, $y'(0) = 1$,

where g = g(t) is a given function.

2. (10 pts) Find the inverse Laplace transform of $\frac{1}{(s^2+1)s^3}$.

3. (10 pts) Solve the integral equation
$$y(t) + \int_{0}^{t} (t-v)y(v) \, dv = t^{2}$$
.

- 4. Solve the following initial value problems:
 - (a) (10 pts) $y'' + 5y' + 6y = e^{-t}\delta(t-2), \quad y(0) = 2, \quad y'(0) = -5.$ (b) (10 pts) $\begin{cases} x' - 3x + 2y = \sin t, & x(0) = 0, \\ 4x - y' - y = \cos t, & y(0) = 0. \end{cases}$ (c) (10 pts) $\begin{cases} x' + y = x, & x(0) = 0, \\ 2x' + y'' = u(t-3), & y(0) = 1, y'(0) = -1. \end{cases}$
- 5. Consider the IVP for y = y(x),

$$y'' + 2y' + y = x^2, \quad y(0) = y'(0) = 0.$$
 (*)

- (a) (5 pts) Use power series in x to solve (*); compute the first four nonzero terms of the series.
- (b) (5 pts) Find the solution y of (*) using the method of undetermined coefficients. Determine the Taylor series of y about $x_0 = 0$. Is this the same series as in (a)?
- 6. (10 pts) Find the first four nonzero terms in the power series solution about $x_0 = 0$ to

$$y'' + (x - 2)y' - y = 0, \quad y(0) = -1, \quad y'(0) = 0.$$

7. (10 pts) Find a lower bound for the radius of convergence of the power series solution about $x_0 = 1$ to

$$(1+x^3)y'' - xy' + 3x^2y = 0.$$

8. (10 pts) Find the first four nonzero terms of the solution to

$$y'' + ty' + e^t y = 0$$
, $y(0) = 0$, $y'(0) = -1$.