Math 334—Assignment 3

1. Using the definition determine the Laplace transform of the function:

$$f(t) = \begin{cases} 1-t, & 0 < t < 1, \\ 0, & 1 < t. \end{cases}$$

Use the Laplace transform table to determine the following transforms:

2. $\mathcal{L}\{t^4 - t^2 - t + \sin(\sqrt{2}t)\}$.

3. $\mathcal{L}\{t \sin^2 t\}$

4. Starting with the transform $\mathcal{L}\{1\}(s) = 1/s$, use the formula for the derivative of the Laplace transform to show that $\mathcal{L}\{t^n\} = n!/s^{n+1}$.

Determine the inverse Laplace transform of the functions:

5.

$$\frac{3}{(2s+5)^3}$$

6.

$$\frac{7s^3 - 2s^2 - 3s + 6}{(s-2)s^3}$$

Solve the given initial value problem using Laplace transforms:

7. $y'' + y = t, \quad y(\pi) = 0, \quad y'(\pi) = 0.$

8. $y'' + ty' - y = 0, \quad y(0) = 0, \quad y'(0) = 3.$

9. Determine the inverse Laplace transform of: $\dfrac{e^{-s}}{s^2+4}$.

10. Solve for the current $I(t)$ governed by the initial-value problem: $I'' + 4I = g(t), \quad I(0) = 1, \quad I'(0) = 3$ where

$$g(t) = \begin{cases} 3\sin t, & 0 < t < 2\pi, \\ 0, & 2\pi < t. \end{cases}$$

First write $g(t)$ in terms of unit step functions.

11. Solve the initial-value problem: $y'' + 4y' + 4y = u(t-\pi) - u(t-2\pi), \quad y(0) = 0, \quad y'(0) = 0.$