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Global solvability and stability of cognitive consumer-resource model with nonlocal usage of memory

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ABSTRACT

In this paper, we formulate a consumer-resource system incorporating dynamic cognitive mapping and nonlocal memory integration. The model represents resource perception using a spatial convolution kernel, capturing nonlocal interactions between consumers and their environment. We establish the global existence of classical solutions under periodic boundary conditions in two spatial dimensions and demonstrate the global stability of homogeneous steady states within specific parameter regimes. Additionally, numerical simulations are conducted to explore the influence of the perception radius R on system dynamics. Our results reveal that the perception radius plays a pivotal role in inducing phase transitions from uniform states to complex spatiotemporal patterns, underscoring the significance of cognitive sensing scales in ecological self-organization.

1. Introduction

The ability of organisms to perceive and process environmental information through spatiotemporal integration of sensory inputs fundamentally shapes ecological interactions and population dynamics. Non-local sensing mechanisms, particularly those involving cognitive mapping and memory-based resource utilization, play a pivotal role in mediating consumer-resource interactions across spatial scales. Such cognitive processes enable organisms to integrate past experiences with current environmental cues through memory kernels, forming dynamic perceptual maps that guide foraging strategies and habitat selection. For instance, many avian species employ spatial memory to optimize foraging patterns, balancing current resource availability

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with historically productive locations [2,5,13,33]. Similarly, fish schools demonstrate collective memory integration, where individuals combine personal experience with group movement history to navigate resource gradients [7,17,21]. These cognitive mechanisms transcend simple stimulus-response behaviors by incorporating temporal memory effects and spatial convolution of environmental signals - precisely the features captured in our proposed mathematical framework.

Non-local advection terms are often employed in mathematical models to describe these interactions, playing a central role in understanding the dynamics of biological systems [10,20,22,27,29,31,32]. These terms are crucial for modeling a wide range of phenomena, including predator-prey dynamics, animal swarming, and cellular navigation. In ecology, animals utilize non-local sensing to make critical decisions, such as avoiding predators, locating prey, or forming aggregations like swarms, flocks, or herds [6,12,14,18,19,26]. Similarly, in cell biology, cells extend protrusions to probe their environment, enabling them to navigate and respond to external stimuli. Chemotaxis—the process by which organisms move along chemical gradients—is another example of non-local advective behavior. These interactions occur across multiple spatial scales, influencing population distributions and leading to complex patterns such as aggregation, segregation, and mixing [15]. Understanding these patterns is essential for predicting species responses to environmental changes.

Recent advances in mathematical modeling have sought to incorporate cognitive processes—such as perception, memory, and learning—into deterministic frameworks, particularly through partial differential equations (PDEs). These models aim to capture the nuanced movement behaviors of animals, which are distinct from the simple particle movements observed in physical or chemical systems. The integration of cognitive dynamics into ecological models has opened new avenues for understanding species distribution and movement patterns. Several studies have explored the intersection of cognitive dynamics and resource interactions. For instance, Wang et al. [25] conducted numerical simulations to analyze cognitive consumer-resource spatiotemporal dynamics with non-local perception, highlighting the importance of understanding system dynamics for effective resource management. Wang et al. [28] systematically investigated critical challenges in partial differential equation (PDE) frameworks applied to knowledge-driven animal movement patterns. Their research particularly highlighted two underexplored yet crucial aspects: non-local sensory perception mechanisms and cognitive spatial mapping processes. Additionally, Pu et al. [23] explored the impact of spatial memory on spatiotemporal patterns in a predator-prey system, while Song et al. [24] investigated memory-based movement with spatiotemporal distributed delays, underscoring the significance of cognitive mapping in understanding resource dynamics. These studies contribute to the growing body of literature on cognitive consumer-resource systems with non-local perception and dynamic mapping.

Despite these advancements, significant challenges remain in the mathematical modeling of non-local interactions. For example, Carrillo et al. [3] constructed the unique mild solution of a non-local multi-species advection-diffusion model under the assumption that the kernel $G(x-y)$ is twice differentiable and $\nabla G(x-y) \in L^\infty(U)$. Their approach, based on semigroup theory and a contraction mapping argument with Duhamel's formula, yielded globally existing solutions only for $n = 1$. Carrillo [4] further proved the existence of weak solutions for aggregation-diffusion equations with merely bounded non-local interaction potentials. Liu et al. [16] investigated a non-local single-species reaction-diffusion-advection model, establishing the existence and uniqueness of a Hölder continuous weak solution in one spatial dimension under general conditions, including discontinuous kernels such as the top-hat detection kernel.

Building upon these cognitive foundations and mathematical challenges, we propose a novel framework that extends the analytical tractability to biologically realistic 2D domains. Our model is built on a two-stage cognitive process, which clarifies our use of the term "nonlocal usage of memory." First, the variable $z(x,t)$ represents a dynamic memory trace of resource availability, not just an instantaneous perception. Its governing equation, $\frac{\partial z}{\partial t} = bv - \gamma z$, models memory formation (accumulation based on resource v) and decay (fading at rate γ). Therefore, $z(x,t)$ acts as a temporally integrated cognitive map of the past resource landscape. Second, the model incorporates the nonlocal usage of this memory. A consumer's movement decision is driven by $\nabla\hat{z}$, where $\hat{z}(x,t) = \int_U G(x-y)z(y,t)dy$. This convolution signifies that a consumer at location x does not rely solely on its memory of that single point. Instead, it accesses and spatially integrates its memory map over a surrounding area defined by the kernel $G(x-y)$. We refer to $G(x-y)$ as the memory usage kernel and its effective radius R as the memory usage range. This mechanism where an organism integrates spatially distributed memories to guide its behavior—is what we term the "nonlocal usage of memory."

In this context, we present a consumer-resource model described by the following system of equations:

$$\begin{cases} \frac{\partial u}{\partial t} = d_1\Delta u - \chi\nabla\left(u\nabla\hat{z}\right) + \frac{c\beta uv}{a+v} - \theta u - du^2, & x\in U, \quad t>0, \\ \frac{\partial v}{\partial t} = d_2\Delta v + rv\left(1-\frac{v}{K}\right) - \frac{\beta uv}{a+v}, & x\in U, \quad t>0, \\ \frac{\partial z}{\partial t} = bv - \gamma z, & x\in U, \quad t>0, \end{cases} \tag{1.1}$$

where $u(x,t)$ and $v(x,t)$ represent the consumer and resource densities, respectively. The parameters r, K, β, a, d, and c denote the resource reproduction rate, carrying capacity, consumer growth rate, half-saturation constant, decay rate, and conversion efficiency, respectively. The spatial domain $U = \mathbb{R}^2/L\mathbb{Z}^2 \cong \left(-\frac{L}{2},\frac{L}{2}\right)^2 \subset \mathbb{R}^2$ is the torus of size $L>0$.

The perception function $\hat{z}(x,t)$, which captures the non-local interaction between consumers and their environment, is defined as a spatial convolution of the resource density $z(y,t)$ with a memory usage kernel $G(x-y)$:

$$\hat{z}(x,t) = \int\limits_U z(y,t)G(x-y)dy.$$

The kernel $G(x-y)$ describes the consumer's memory usage range and sensitivity, which can take various forms, such as uniform, Gaussian, or exponential distributions. For biological relevance, our hypotheses on the kernels $G(x-y)$ are as follows:

- (H1) $G(x-y) \in L^1(U)\cap L^\infty(U)$, which implies that there exists a positive constant G_0 such that $\max\{\|G(x-y)\|_{L^1}, \|G(x-y)\|_{L^\infty}\} \le G_0$.
- (H2) There exists $C>0$ such that for all smooth $\phi(\cdot)$, $\|\nabla G * \phi\|_{L^1(U)} \le C\|\phi\|_{L^1(U)}$.
- (H3) $G(x-y)>0$, $G(x-y)=G(y-x)$, and $\int_U G(x-y)dy = 1$ for $x,y\in U$.
- (H4) $G(x-y)$ is compactly supported.

Specifically, we consider the following memory usage kernel:

$$G(x-y) := \begin{cases} \frac{1}{\pi R^2}, & \|x-y\|\le R, \\ 0, & \text{otherwise}, \end{cases} \tag{1.2}$$

where $\|x-y\|$ is the Euclidean distance between points x and y, and $R\ge 0$ is the memory usage range parameter.

The initial data $(u_0(x), v_0(x), z_0(x))$ are assumed to satisfy the following conditions:

$$\begin{cases} u_0(x)\in C^{2+\nu}(U) \quad \text{with} \quad u_0(x)>0 \quad \text{in} \quad U, \\ v_0(x)\in C^{2+\nu}(U) \quad \text{with} \quad v_0(x)>0 \quad \text{in} \quad U, \\ z_0(x)\in C^{2+\nu}(U) \quad \text{with} \quad z_0(x)>0 \quad \text{in} \quad U, \end{cases} \tag{1.3}$$

where $0<\nu<1$ denotes the Hölder exponent.

Despite the absence of a rigorous well-posedness theory for systems employing the top-hat kernel, researchers can analytically investigate many models with non-local animal interactions through linear stability analysis of pattern formation [25]. One of the primary challenges in this field stems from the inherent mathematical complexity of non-local operators in continuum models. Establishing the global existence and stability of solutions necessitates rigorous analytical techniques and advanced numerical methodologies. The principal objectives of this study are dual in nature: first, to establish the global existence of classical solutions under periodic boundary conditions in two dimensions, and second, to rigorously demonstrate the global stability of homogeneous steady states under specific parameter regimes. This work resolves an open problem posed by Wang et al. [28] by developing a comprehensive well-posedness theory applicable to a broad class of kernels, including (1.2).

The remainder of this paper is organized as follows. In Section 2, we show the local existence of solutions and prove some basic properties of solutions. In Section 3, the boundedness and global existence of the time-varying solution of the system (1.1) are derived. In Section 4, we prove the global stability of constant steady states under certain conditions. In Section 5, by numerical simulations we investigate the role of the detection scale R in generating spatiotemporal patterns using the top-hat detection function as a case study. In Section 6, we give a brief conclusion.

2. Local existence

This section establishes the existence of local solutions for the system formulated in (1.1) through the application of established theoretical frameworks for quasilinear parabolic systems, as developed by Amann [1] for quasilinear parabolic systems.

Lemma 2.1 *(Local existence). Assume that the initial conditions satisfy* (1.3). *There is a sufficiently small* $T_{\max} > 0$ *such that the problem* (1.1) *has a unique classical solution*

$$(u, v, z) \in \left(C^{2+\alpha, 1+\frac{\alpha}{2}}\left(U \times [0, T_{\max})\right)\right)^3, \tag{2.1}$$

with

$$u > 0, v > 0, z > 0 \quad (x, t) \in U \times (0, T_{\max}), \tag{2.2}$$

and such that

$$\text{either } T_{\max} = \infty \quad \text{or } \limsup_{t \nearrow T_{\max}} \left(\|u(\cdot, t)\|_{C^{2+\beta}(U)} + \|v(\cdot, t)\|_{C^{2+\beta}(U)} + \|z(\cdot, t)\|_{C^{2+\beta}(U)}\right) = \infty \text{ for all } \beta \in (0, 1). \tag{2.3}$$

Proof. Let $T \in (0, 1)$ be a constant to be determined later. We define the Banach space X as the space of functions u endowed with the finite norm $\|u\|_{C^{1,0}(U \times [0,T])}$. Within this framework, we introduce the closed convex subset $S_T \subset X$ as follows:

$$S_T := \left\{u \in X \middle|\ u \geq 0, \|u\|_{C^{1,0}(U \times [0,T]} \leq M\right\},$$

where the constant M is defined by

$$M = \|u_0\|_{C^{2+\alpha}(U)} + 1. \tag{2.4}$$

For any $\bar{u} \in S_T$, we consider the following initial-boundary value problem

$$\begin{cases} v_t = d_2 \Delta v + rv\left(1 - \frac{v}{K}\right) - \frac{\beta \bar{u} v}{a+v}, \\ v(x,0) = v_0(x), \end{cases} \tag{2.5}$$

subject to periodic boundary conditions.

Applying the comparison principle for parabolic equation to problem (2.5), we obtain

$$\|v\|_{L^\infty(U\times(0,T))} \le \max\{K, \|v_0\|_{L^\infty(U)}\} := M_v. \tag{2.6}$$

We now fix any $p > 4$ and utilize the fact that $\bar{u} \in S_T$. Applying standard parabolic L^p regularity theory to the linear problem (2.5), we obtain

$$\|v\|_{W_p^{2,1}(U\times(0,T))} \le c_1 \left(\|v_0\|_{W^{2,p}(U)} + \|F(v,\bar{u})\|_{L^p(U\times(0,T))}\right),$$

where $F(v,\bar{u}) = rv\left(1 - \frac{v}{K}\right) - \frac{\beta \bar{u} v}{a+v}$.

We estimate the nonlinear term the nonlinear term $F(v,\bar{u})$ as follows

$$\|F(v,\bar{u})\|_{L^\infty(U\times(0,T))} \le r\|v\|_{L^\infty(U\times(0,T))} + \beta\|\bar{u}\|_{L^\infty(U\times(0,T))} \le rM_v + \beta M,$$

where $M = \|\bar{u}\|_{L^\infty(U\times(0,T))}$.

Combining the parabolic L^p theory and the estimates for $F(v,\bar{u})$, we obtain

$$\|v\|_{W_p^{2,1}(U\times(0,T))} \le c_1 \left(\|v_0\|_{W^{2,p}(U)} + rM_v + \beta M\right) := c_2, \tag{2.7}$$

where c_1 and c_2 are constants derived from the parabolic L^p theory. All subsequent constants c_3, c_4, $\cdots$ are positive and depend on M but not on T.

Since $p > 2$, the Sobolev embedding theorem implies that $W_p^{2,1}(U \times (0,T))$ can be continuously embedded into $C^{1,0}(U \times [0,T])$. Consequently, there exists a constant $c_3 > 0$ such that

$$\|\nabla v\|_{L^\infty(U\times(0,T))} \le c_3\|v\|_{W_p^{2,1}(U\times(0,T))}, \|\Delta v\|_{L^\infty(U\times(0,T))} \le c_3\|v\|_{W_p^{2,1}(U\times(0,T))}.$$

Combining this with the estimate for $\|v\|_{W_p^{2,1}(U\times(0,T))}$, we obtain

$$\|\nabla v\|_{L^\infty(U\times(0,T))} \le c_4 \text{ and } \|\Delta v\|_{L^\infty(U\times(0,T))} \le c_4,$$

combining this with the maximum principle, we have

$$v \in C^{1,0}(\bar{U} \times [0,T]) \quad \text{and} \quad v \ge 0. \tag{2.8}$$

Next, we consider

$$\begin{cases} z_t = bv - \gamma z, & x \in U, t > 0, \\ z(x,0) = z_0(x), & x \in \partial U. \end{cases} \tag{2.9}$$

It is evident that

$$z(x,t) = e^{-\gamma t}\left[z_0(x) + b\int_0^t e^{\gamma\tau}\bar{v}(\tau)\mathrm{d}\tau\right], \quad x \in U,\ t \in \times[0,T). \tag{2.10}$$

Therefore, with $T \in (0,1)$ we have

$$\|z\|_{L^\infty(U\times(0,T))} \leq \|z_0\|_{L^\infty} + \frac{bM_v}{\gamma} := c_5. \tag{2.11}$$

To derive appropriate estimates for ∇z, we differentiate (2.11) and observe that ∇z satisfies the initial-value problem

$$\begin{cases} (\nabla z)_t = -\gamma \nabla z + b\nabla v, & \text{in } U \times (0,T), \\ \nabla z(\cdot,0) = \nabla z_0, & \text{in } U. \end{cases} \tag{2.12}$$

Therefore,

$$\begin{aligned} \big\|\nabla z(\cdot,t)\big\|_{L^\infty(U)} \leq& \big\|\nabla z_0\big\|_{L^\infty(U)} e^{-\gamma T} + bc_4 T e^{bc_4 T} \\ \leq& \big\|z_0\big\|_{W^{1,\infty}(U)} + bc_4 e^{bc_4} := c_6, \text{ for all } t \in (0,T). \end{aligned} \tag{2.13}$$

Similarly, we can obtain

$$\big\|\Delta z(\cdot,t)\big\|_{L^\infty(U)} \leq \big\|\Delta z_0\big\|_{L^\infty(U)} + \frac{\gamma c_4}{b} := c_7. \tag{2.14}$$

Thus, using (2.8), (2.12) and (2.13) we see that in fact

$$\big\|z\big\|_{C^{1,1}(\bar U\times[0,T])} \leq c_8. \tag{2.15}$$

Finally, we analyze the semilinear parabolic equation

$$\begin{cases} u_t = d_1\Delta u + f_1(x,t)\cdot\nabla u + f_2(x,t)u, & x\in U, t>0, \\ u(x,0) = u_0(x), & x \in U, \end{cases} \tag{2.16}$$

with

$$\begin{aligned} &f_1 := -\chi\nabla(G*z), \\ &f_2 := -\chi\Delta(G*z) + \tfrac{c\beta v}{a+v} - d\bar u - \theta. \end{aligned}$$

By (2.13) and (2.14), and Young's convolution inequality, we can find some $c_9 > 0$ such that

$$\begin{aligned} \big\|\nabla(G*z)\big\|_{L^\infty} = \big\|G*\nabla z\big\|_{L^\infty} \leq \big\|G(x-y)\big\|_{L^1}\big\|\nabla z\big\|_{L^\infty} \leq c_9, \\ \big\|\Delta(G*z)\big\|_{L^\infty} = \big\|G*\Delta z\big\|_{L^\infty} \leq \big\|G(x-y)\big\|_{L^1}\big\|\Delta z\big\|_{L^\infty} \leq c_9. \end{aligned} \tag{2.17}$$

Since $\bar u \in S_T$, by (2.6), and (2.17) we obtain $c_{10} > 0$ fulfilling

$$\big\|f_1\big\|_{L^\infty} \leq c_{10}, \big\|f_2\big\|_{L^\infty} \leq c_{10}, \text{ for all } \ t \in (0,T).$$

Fix $p > 4$, using this along with parabolic L^p-theory applied to the linear problem (2.16) we find $c_{11} > 0$ such that

$$\big\|u\big\|_{W_p^{2,1}(U\times(0,T))} \leq c_{11}. \tag{2.18}$$

Since $0 < T < 1$, we apply a Sobolev embedding inequality to derive

$$\|u\|_{C^{1+s,\frac{1+s}{2}}(U\times[0,T])} \leq c_{12}\|u\|_{W_p^{2,1}(U\times(0,T))} \tag{2.19}$$

with $s := 1 - \frac{4}{p}$, where $p > 4$ and some $c_{12} > 0$.

We next note that (2.19) also imply that

$$\|u\|_{C^{1,\frac{\gamma}{2}}(U\times[0,T])} \leq c_{13}. \tag{2.20}$$

This yields that

$$\begin{aligned}\|u\|_{C^{1,0}(U\times[0,T])} =& \|(u-u_0)+u_0\|_{C^{1,0}(U\times[0,T])}\\ \leq& \|u-u_0\|_{C^{1,0}(U\times[0,T])} + \|u_0\|_{C^{1,0}(U\times[0,T])}\\ \leq& T^{\frac{s}{2}}\|u\|_{C^{1,\frac{s}{2}}(U\times[0,T])} + \|u_0\|_{W^{1,\infty}(U)}\\ \leq& T^{\frac{s}{2}}c_{13} + \|u_0\|_{W^{1,\infty}(U)}.\end{aligned} \tag{2.21}$$

By selecting $T \in (0,1)$ sufficiently small to satisfy

$$T^{\frac{\gamma}{2}}c_5 < 1,$$

then from (2.21) we derive that

$$\|u\|_{C^{1,0}(U\times[0,T])} + \|v\|_{C^{1,0}(U\times[0,T])} \leq 1 + \|u_0\|_{W^{1,\infty}(U)} + \|v_0\|_{W^{1,\infty}(U)} \leq M.$$

Having established uniform bounds for u, v, z, we demonstrate that the mapping

$$\mathcal{F} : S_T \to C^{1,0}(U\times[0,T)), \quad \bar{u} \mapsto u,$$

defined by equations (2.9) and (2.16), is well-defined, and $\mathcal{F}$ maps S_T into itself. Furthermore, using standard techniques, it can be shown that $\mathcal{F}$ is contractive on S_T for sufficiently small T. Thus, by the contraction mapping principle, $\mathcal{F}$ possesses a unique fixed point u in S_T. Additionally, the non-negativity of u, i.e., $u \geq 0$, follows directly from the maximum principle.

Using standard bootstrapping arguments grounded in elliptic and parabolic Schauder estimates (see [11]), along with the regularity and first-order compatibility conditions provided by (1.3), we establish that the solution (u,v,z) fulfills the regularity properties outlined in (2.1). The positivity conditions in (2.2) are immediate from the parabolic strong maximum principle. Moreover, the alternative (2.3) arises naturally since T is chosen based solely on the constant M defined in (2.4). □

Before proving that $T_{\max} = \infty$, we first relax the extensibility criterion in (2.3) as follows.

Lemma 2.2. *For u_0, v_0, z_0 satisfying (1.3), the solution (u,v,z) of (1.1) constructed in Lemma 2.1 satisfies*

$$\textit{either} \quad \limsup_{t\nearrow T_{max}} \|u(\cdot,t)\|_{L^\infty(U)} = \infty \quad \textit{or} \quad T_{max} = \infty. \tag{2.22}$$

Proof. Assume, by contradiction, that $T_{\max} < \infty$ and there exists $c_1 > 0$ satisfying

$$\|u(\cdot,t)\|_{L^\infty(U)} \leq c_1 \quad \text{for all } t \in (0, T_{\max}). \tag{2.23}$$

Then, using the same methods applied in the proof of Lemma 2.1, we know that there exists some c_2 such that

$$\|v\|_{W^{2,\infty}(U)} \leq c_2, \quad \|z\|_{W^{2,\infty}(U)} \leq c_2 \quad \text{for all } t \in (0, T_{\max}). \tag{2.24}$$

Consequently, with the Sobolev embedding theorem in the two-dimensional setting, this shows

$$\|v\|_{C^{1+\alpha,\frac{1+\alpha}{2}}(U\times[0,T_{\max}])} \leq c_3, \quad \|z\|_{C^{1+\alpha,\frac{1+\alpha}{2}}(U\times[0,T_{\max}])} \leq c_3. \tag{2.25}$$

The equation for u can be rewritten as

$$u_t = \Delta u + f(x,t) \cdot \nabla u + g(x,t)u \quad \text{in } U \times (0, T_{\max}), \tag{2.26}$$

where

$$f := -\chi\nabla(G * z),\ g := -\chi\Delta(G * z) + \tfrac{c\beta v}{a+v} - du - \theta. \tag{2.27}$$

From (2.23) and Young's convolution inequality, it follows that there exists $c_4 > 0$ satisfying

$$\|f(\cdot,t)\|_{L^\infty(U)}, \|g(\cdot,t)\|_{L^\infty(U)} \leq c_4 \quad \text{for all } t \in (0, T_{\max}).$$

This in conjunction with parabolic L^p-theory, (2.23) and (2.26) entails that there exists some $c_5 > 0$ such that

$$\|u\|_{W_p^{2,1}(U\times(0,T_{\max}))} \leq c_5. \tag{2.28}$$

Combined with the Sobolev embedding theorem in two dimensions, this yields

$$\|u\|_{C^{1+\alpha,\frac{1+\alpha}{2}}(U\times[0,T_{\max}])} \leq c_7. \tag{2.29}$$

Combining with (2.25) and (2.29), we obtain

$$\|u\|_{C^{2+\beta,0}(\bar{U}\times[0,T_{\max}])} \leq c_{12} \quad \text{and} \quad \|z\|_{C^{2+\beta,0}(\bar{U}\times[0,T_{\max}])} \leq c_{12}. \tag{2.30}$$

This contradicts the extensibility criterion (2.3) in Lemma 2.1, thus establishing $T_{\max} = \infty$. □

Lemma 2.3. *The unique solution (u, v, z) of system (1.1) satisfies*

$$\|v(\cdot,t)\|_{L^\infty} \leq \max\left\{\|v_0\|_{L^\infty}, K\right\} := M_1,\ \|z(\cdot,t)\|_{L^\infty} \leq \max\left\{\|z_0\|_{L^\infty}, \frac{bK}{\gamma}\right\} := M_2, \quad \textit{for all } x \in U \textit{ and } t > 0. \tag{2.31}$$

Proof. Applying the parabolic comparison principle to the second and third equations of (1.1), we derive $u, v > 0$ for all $t > 0$. □

For subsequent applications, we recall the standard L^p-L^q estimates for the heat semigroup with periodic boundary conditions.

Lemma 2.4. *Let the heat semigroup $(e^{td\Delta})_{t\geq 0}$ be defined on a periodic domain U. Here, $\lambda_1 > 0$ represents the first nonzero eigenvalue of the operator $-\Delta$ under periodic boundary conditions, with $d > 0$ being a positive constant. There exist constants $\xi_1, \ldots, \xi_4$, determined only by U, with the following properties:*

(i) *For* $1 \le q \le p \le \infty$ *and any* $w \in L^q(U)$ *satisfying* $\int_U w = 0$, *the following estimate holds:*

$$\|e^{td\Delta} w\|_{L^p(U)} \le \xi_1 \left(1 + t^{-\frac{n}{2}(\frac{1}{q}-\frac{1}{p})}\right) e^{-d\lambda_1 t} \|w\|_{L^q(U)}, \quad \textit{for all } t > 0. \tag{2.32}$$

(ii) *For* $1 \le q \le p \le \infty$ *and any* $w \in L^q(U)$, *the gradient of the solution satisfies:*

$$\|\nabla e^{td\Delta} w\|_{L^p(U)} \le \xi_2 \left(1 + t^{-\frac{1}{2}-\frac{n}{2}(\frac{1}{q}-\frac{1}{p})}\right) e^{-d\lambda_1 t} \|w\|_{L^q(U)}, \quad \textit{for all } t > 0. \tag{2.33}$$

(iii) *For* $2 \le p < \infty$ *and any* $w \in W^{1,p}(U)$, *the gradient of the solution satisfies:*

$$\|\nabla e^{td\Delta} w\|_{L^p(U)} \le \xi_3 e^{-d\lambda_1 t} \|\nabla w\|_{L^p(U)}, \quad \textit{for all } t > 0. \tag{2.34}$$

(iv) *For* $1 < q \le p < \infty$ *and any* $w \in (C_0^\infty(U))^n$, *the divergence of the solution satisfies:*

$$\|e^{td\Delta}(\nabla \cdot w)\|_{L^p(U)} \le \xi_4 \left(1 + t^{-\frac{1}{2}-\frac{n}{2}(\frac{1}{q}-\frac{1}{p})}\right) e^{-d\lambda_1 t} \|w\|_{L^q(U)}, \quad \textit{for all } t > 0. \tag{2.35}$$

Moreover, the operator $e^{t\Delta}\nabla\cdot$ *admits a unique extension to an operator from* $L^q(U)$ *to* $L^p(U)$, *with its norm controlled by* (2.35).

Remark 2.1. The result for the Neumann heat semigroup, along with its detailed proof, has been presented in Reference [30]. While Lemma 2.4 is concerned with periodic boundary conditions, its proof follows a completely analogous approach to that in Reference [30]. We omit the proof for brevity.

The following Gagliardo-Nirenberg inequality, which is crucial for our subsequent analysis, has its proof detailed in [9].

Lemma 2.5 *(Gagliardo-Nirenberg inequality). Let* $\Omega \subset R^n$ *be a bounded domain with a smooth boundary, and let* $\varphi \in L^r(\Omega) \cap W^{k,q}(\Omega)$ *with* $1 \le q, r \le \infty$ *and* $k > 0$. *Then there exists a constant* $C_{GN} > 0$ *such that*

$$\|\varphi\|_{L^p(\Omega)} \le C_{GN} \left(\|D^k \varphi\|^{\lambda}_{L^q(\Omega)} \|\varphi\|^{1-\lambda}_{L^r(\Omega)} + \|\varphi\|_{L^r(\Omega)}\right),$$

where $\lambda \in (0,1)$ *satisfies the relation* $\frac{1}{p} = \lambda(\frac{1}{q} - \frac{k}{n}) + \frac{1}{r}(1-\lambda)$.

3. Boundedness and global existence

This section is devoted to the rigorous derivation of global a priori estimates for the solutions of system (1.1), which serves as a crucial foundation for extending the existence of solutions from local to global domains. Under the hypothesis that $T_{\max} < \infty$ and by leveraging the result stated in (2.22), we derive uniform bounds on the L^∞-norm of $u(\cdot,t)$ over the interval $(0, T_{\max})$. In this context, the symbols c_i or C_i $(i = 1, 2, 3, \ldots)$ denote arbitrary positive constants whose values may be adjusted according to the specific requirements of the analysis.

We verify the following essential properties.

Lemma 3.1. *Let* (u, v, z) *be a solution to system* (1.1) *defined on* $U \times (0, T_{max})$. *Then there exist positive constants* C_1 *and* C_2, *independent of* t, *satisfying the following uniform estimates*

$$\|u(\cdot,t)\|_{L^1(U)} \le C_1 \quad \textit{for all } t \in (0, T_{max}), \tag{3.1}$$

and

$$\int_{t}^{t+\tau}\int_{U} u^2(x,s)\,dx\,ds \le C_2 \quad \textit{for } \tau = \min\left\{1, \tfrac{1}{2}T_{\max}\right\}. \tag{3.2}$$

Proof. By integrating the first equation in (1.1) over the domain U and using the divergence theorem on the diffusion and cross-diffusion terms (which vanish due to periodic boundary conditions), we get

$$\frac{d}{dt}\int_U u\,dx = \int_U \left(\frac{c\beta uv}{a+v} - \theta u - du^2\right) dx. \tag{3.3}$$

Similarly, multiplying the second equation by c and integrating gives

$$\frac{d}{dt}\int_U cv\,dx = \int_U \left(crv\left(1-\frac{v}{K}\right) - \frac{c\beta uv}{a+v}\right) dx. \tag{3.4}$$

Adding these two equations, the interaction terms cancel, yielding

$$\frac{d}{dt}\left(\int_U (u+cv)\,dx\right) = \int_U \left(crv\left(1-\frac{v}{K}\right) - \theta u - du^2\right) dx. \tag{3.5}$$

Let $Y(t) = \int_U (u+cv)dx$. To form an inequality suitable for Grönwall's inequality, we add $\theta Y(t) = \theta\int_U (u+cv)dx$ to both sides of (3.5)

$$\frac{d}{dt}Y(t) + \theta Y(t) = \int_U \left(crv - \frac{crv^2}{K} - \theta u - du^2\right) dx + \int_U (\theta u + c\theta v)dx.$$

Simplifying the right-hand side, we obtain

$$\frac{d}{dt}Y(t) + \theta Y(t) + d\int_U u^2 dx = \int_U \left(c(r+\theta)v - \frac{crv^2}{K}\right) dx. \tag{3.6}$$

Building upon the a priori bound $0 \le v \le M_1$ and noting that $-\frac{cr}{K}v^2 \le 0$, we can establish an upper bound for the right-hand side:

$$\int_U \left(c(r+\theta)v - \frac{crv^2}{K}\right) dx \le \int_U c(r+\theta)v\,dx \le c(r+\theta)M_1|U|.$$

Therefore, inequality (3.6) implies:

$$\frac{d}{dt}Y(t) + \theta Y(t) \le c(r+\theta)M_1|U|.$$

Applying Grönwall's inequality to this differential inequality immediately yields the uniform bound for $Y(t)$, which proves (3.1).

Furthermore, by integrating (3.6) over the interval $(t, t+\tau)$, we have

$$Y(t+\tau) - Y(t) + \theta\int_t^{t+\tau} Y(s)ds + d\int_t^{t+\tau}\int_U u^2 dxds = \int_t^{t+\tau}\int_U \left(c(r+\theta)v - \frac{crv^2}{K}\right) dxds.$$

Since $Y(t)$ is bounded by a constant C_1 (from (3.1)) and the right-hand side is also bounded above by a constant, we can rearrange to find

$$d\int_t^{t+\tau}\int_U u^2 dxds \le Y(t) - Y(t+\tau) + \int_t^{t+\tau} \left(c(r+\theta)M_1|U| - \theta Y(s)\right) ds.$$

Since all terms on the right-hand side are uniformly bounded for any τ, we conclude that there exists a constant C_2 such that the integral of u^2 is bounded over any time interval of fixed length, which yields (3.2). □

Having established basic mass conservation properties, we next investigate the spatial regularity of resource distribution. The following gradient estimates for v will form the foundation for subsequent analysis of cross-diffusion terms.

Lemma 3.2. *Let (u, v, z) be a solution to system (1.1) defined on $U \times (0, T_{max})$. There exist positive constants $C_3 > 0$ and $C_4 > 0$ such that the following estimates hold:*

$$\|\nabla v(\cdot,t)\|_{L^2(U)} \le C_3 \quad \text{for all } t \in (0, T_{\max}) \tag{3.7}$$

and

$$\int_t^{t+\tau} \|\Delta v(\cdot,s)\|^2_{L^2(U)}\, ds \le C_4 \quad \text{for all } t \in (0, T_{\max}-\tau). \tag{3.8}$$

Proof. Consider the second equation in (1.1). Through the duality pairing with $-\Delta v$ and subsequent integration by parts, we obtain the fundamental energy identity:

$$\begin{aligned}
\frac{1}{2}\frac{d}{dt}\int_U |\nabla v|^2 dx + d_2\int_U |\Delta v|^2 dx &= -\int_U \left(rv(1-\frac{v}{K}) - \frac{\beta uv}{a+v}\right)\Delta v dx \\
&\le \int_U (rM_1 + \frac{r}{K}M_1^2 + \beta u)|\Delta v| dx \\
&\le \frac{(rM_1 + \frac{r}{K}M_1^2)^2}{4d_2}|U| + \frac{d_2}{2}\int_U |\Delta v|^2 dx + \frac{\beta^2}{d_2}\int_U u^2 dx.
\end{aligned} \tag{3.9}$$

This yields

$$\frac{d}{dt}\int_U |\nabla v|^2 dx + d_2\int_U |\Delta v|^2 dx \le \frac{(rM_1 + \frac{r}{K}M_1^2)^2}{2d_2}|U| + \frac{2\beta^2}{d_2}\int_U u^2 dx. \tag{3.10}$$

Implementing the Gagliardo-Nirenberg inequality in 2D domains, and noting that $\|v\|_{L^2} \le M_1|U|^{1/2}$, this interpolates to:

$$\|\nabla v\|^2_{L^2} \le C_{GN}(\|v\|^2_{L^2} + \|\Delta v\|_{L^2}\|v\|_{L^2}) \le \frac{d_2}{2}\|\Delta v\|^2_{L^2} + c_1. \tag{3.11}$$

Substituting (3.11) into (3.10) produces

$$\frac{d}{dt}\|\nabla v(t)\|_{L^2}^2 + \|\nabla v(t)\|_{L^2}^2 + \frac{d_2}{2}\|\Delta v\|_{L^2}^2 \leq c_2\|u\|_{L^2}^2 + c_3. \tag{3.12}$$

Applying Grönwall's inequality in conjunction with the mass control (3.2) yields the uniform gradient bound (3.7). Temporal integration over sliding windows $(t, t+\tau)$ provides the estimate (3.8). $\square$

The gradient bounds for v naturally lead us to examine the memory variable z. Since z is governed by an ODE with v as input, we can leverage these v estimates to control the z dynamics.

Lemma 3.3. *Let (u, v, z) be a solution to system (1.1) defined on $U \times (0, T_{max})$. There exist positive constants $C_5 > 0$ and $C_6 > 0$ such that the following estimates hold:*

$$\|\nabla z(\cdot, t)\|_{L^2(U)} \leq C_5 \quad \textit{for all } t \in (0, T_{\max}) \tag{3.13}$$

and

$$\int_t^{t+\tau}\int_U |\Delta z(x,s)|^2\, dx\, ds \leq C_6 \quad \textit{for all } t \in (0, T_{\max} - \tau). \tag{3.14}$$

Proof. First, we differentiate the third equation in system (1.1), one has

$$(\nabla z)_t = b\nabla v - \gamma\nabla z. \tag{3.15}$$

Multiplying (3.15) with ∇z, and integrate over U, we obtain

$$\begin{aligned} \frac{1}{2}\frac{d}{dt}\int_U |\nabla z|^2 dx =& b\int_U \nabla v \nabla z dx - \gamma\int_U |\nabla z|^2 dx \\ \leq& \frac{b}{2\gamma}\int_U |\nabla v|^2 dx - \frac{\gamma}{2}\int_U |\nabla z|^2 dx. \end{aligned} \tag{3.16}$$

It is noted that (3.7), and by the Grönwall's inequality, we obtain (3.13).

Differentiating Eq. (3.15), we have

$$\Delta z_t = b\Delta v - \gamma\Delta z. \tag{3.17}$$

Multiplying (3.17) by $2\Delta z$, by (3.8) and using Young inequality we have

$$\begin{aligned} (|\Delta z|^2)_t =& 2b\Delta v \nabla z - 2\gamma|\Delta z|^2 \\ \leq& -\gamma|\Delta z|^2 + \frac{b^2}{\gamma}|\Delta v|^2. \end{aligned} \tag{3.18}$$

Therefore,

$$\begin{aligned} \frac{d}{dt}\int_t^{t+\tau}\int_U |\Delta z|^2 dx ds \leq& \frac{b^2}{\gamma}\int_t^{t+\tau}\int_U |\Delta v|^2 dx ds - \gamma\int_t^{t+\tau}\int_U |\Delta z|^2 dx ds \\ \leq& \frac{b^2 C_4}{\gamma} - \gamma\int_t^{t+\tau}\int_U |\Delta z|^2 dx ds, \end{aligned}$$

which leads to

$$\int_t^{t+\tau}\int_U |\Delta z|^2 dxds \le C_6. \quad \square$$

With first-order spatial derivatives under control, we now return to the consumer density u. The following L^2 estimate will enable us to handle nonlinear terms through energy methods and Sobolev embeddings.

Lemma 3.4. *Let (u, v, z) be a solution to system (1.1) defined on $U \times (0, T_{max})$ with initial data (u_0, v_0, z_0) satisfying (1.3). There exists a positive constant $C_7 > 0$, such that the following L^2-estimate holds:*

$$\|u(\cdot,t)\|_{L^2(U)} \le C_7 \quad \text{for all } t \in (0, T_{max}). \tag{3.19}$$

Proof. Multiplying the governing equation for u in system (1.1) by $2u$ and applying Young's inequality yields

$$\begin{aligned}
&\frac{d}{dt}\int_U u^2 dx + 2d_1\int_U |\nabla u|^2 dx + 2d\int_U u^3 dx \\
&= 2\chi\int_U u\nabla u\nabla(G*z)dx + 2\int_U \Big(\frac{c\beta u^2 v}{a+v} - \theta u^2\Big)dx \\
&\le 2\chi\int_U \Big(\frac{d_1|\nabla u|^2}{2\chi} + \frac{\chi u^2|\nabla(G*z)|^2}{2d_1}\Big)dx + 2c\beta\int_U u^2 dx \\
&\le d_1\int_U |\nabla u|^2 dx + \frac{\chi^2}{d_1}\|u\|_{L^4}^2\|\nabla(G*z)\|_{L^4}^2 + 2c\beta\|u\|_{L^2}^2.
\end{aligned} \tag{3.20}$$

Next, we estimate $\|u\|_{L^4}^2$ and $\|\nabla(G*z)\|_{L^4}^2$ respectively. By the two dimensions ($n = 2$) Gagliardo-Nirenberg inequality, we have

$$\|u\|_{L^4}^2 \le C_8(\|u\|_{L^2}^2 + \|\nabla u\|_{L^2}\|u\|_{L^2}). \tag{3.21}$$

Thus, applying Young's convolution inequality and using (3.13) along with hypothesis (H1), we obtain

$$\|\nabla(G*z)\|_{L^4}^2 = \|G*\nabla z\|_{L^4}^2 \le \|G(x-y)\|_{L^{\frac{4}{3}}}^2 \|\nabla z\|_{L^2}^2 \le G_0^{\frac{3}{2}} C_5^2. \tag{3.22}$$

Combining (3.21) with (3.22), we have

$$\begin{aligned}
\frac{\chi^2}{d_1}\|u\|_{L^4}^2\|\nabla(G*z)\|_{L^4}^2 \le &\frac{\chi^2}{d_1}G_0^{\frac{3}{2}}C_5^2C_8(\|\nabla u\|_{L^2}\|u\|_{L^2} + \|u\|_{L^2}^2) \\
\le &d_1\|\nabla u\|_{L^2}^2 + C_9\|u\|_{L^2}^2,
\end{aligned} \tag{3.23}$$

where $C_9 = \frac{2d_1^2}{\chi^4 G_0^3 C_5^4 C_8^2} + \frac{\chi^2}{d_1}G_0^{\frac{3}{2}}C_5^2C_8$.

By substituting (3.23) into (3.20) and applying Young's inequality, we obtain

$$\frac{d}{dt}\|u\|_{L^2}^2 \le C_{10}\|u\|_{L^2}^2. \tag{3.24}$$

From (3.2), for any $t \in (0, T_{\max})$, there exists $t_0 \in [(t-\tau)_+, t)$ satisfying

$$\|u(\cdot, t_0)\|_{L^2}^2 \leq C_{11} := \max\left\{\|u_0\|_{L^2}^2, C_2\right\}. \tag{3.25}$$

Since $t_0 < t \leq t_0 + \tau \leq t_0 + 1$, integrating (3.24) over the interval (t_0, t) yields

$$\|u(\cdot, t)\|_{L^2} \leq \|u(\cdot, t_0)\|_{L^2} e^{C_{10}} := C_7 \tag{3.26}$$

for all $t \in (0, T_{\max})$. □

The L^2 stability of u motivates investigating higher integrability. By employing Gagliardo-Nirenberg inequalities and nonlinear test functions, we next establish an L^3 bound.

Lemma 3.5. *Suppose that* (u_0, v_0, z_0) *satisfies* (1.3)*. Then, there exists a constant* $C_{11} > 0$ *such that*

$$\|\nabla v(\cdot, t)\|_{L^4} + \|\nabla z(\cdot, t)\|_{L^4} \leq C_{11} \quad \forall \quad t \in (0, T_{max}). \tag{3.27}$$

Proof. We begin by reformulating the second equation in (1.1) into the following form:

$$\frac{\partial v}{\partial t} - d_2 \Delta v + v = \Phi(x, t) \tag{3.28}$$

with $\Phi(x,t) := rv\big(1 - \frac{v}{K}\big) - \frac{\beta uv}{a+v} + v$. Then by (3.19) and (2.31), we have

$$\begin{aligned}
\|\Phi(\cdot, t)\|_{L^2} =& \Big\| rv\big(1 - \frac{v}{K}\big) - \frac{\beta uv}{a+v} + v \Big\|_{L^2} \\
\leq& (M_1(r+1) + \beta C_7)|U|^{\frac{1}{2}}.
\end{aligned}$$

From Eq. (3.28), we can get

$$v(\cdot, t) = \int_0^t e^{(t-s)(d_2\Delta - 1)} \Phi(\cdot, s) ds + e^{t(d_2\Delta - 1)} v_0. \tag{3.29}$$

Then using Lemma 2.4, one has

$$\begin{aligned}
\|\nabla v(\cdot, t)\|_{L^4} \leq& \int_0^t \|\nabla e^{(t-s)(d_2\Delta-1)} \Phi(\cdot, s)\|_{L^4} ds + \|\nabla e^{t(d_2\Delta-1)} v_0\|_{L^4} \\
\leq& \|\nabla e^{td_2\Delta} v_0\|_{L^4} e^{-t} + \int_0^t \|e^{(t-s)d_2\Delta} \Phi(\cdot, s)\|_{L^4} ds \\
\leq& 2\xi_2 e^{-d_2\lambda_1 t} \|\nabla v_0\|_{L^4} + \xi_2 \int_0^t \left((t-s)^{-\frac{3}{4}} + 1\right) e^{-\lambda_1 d_2 (t-s)} \|\Phi(\cdot, s)\|_{L^2} ds \\
\leq& 2\xi_2 \|\nabla v_0\|_{L^4} + \xi_2 \left((M_1(r+1) + \beta C_7)|U|^{\frac{1}{2}}\right) \int_0^\infty (z^{-\frac{3}{4}} + 1) e^{-\lambda_1 d_2 z} dz \\
\leq& 2\xi_2 \|\nabla v_0\|_{L^4} + \frac{\xi_2}{d_2\lambda_1} \left((M_1(r+1) + \beta C_7)|U|^{\frac{1}{2}}\right) \left(1 + \Gamma(1/4)(\lambda_1 d_2)^{\frac{3}{4}}\right).
\end{aligned}$$

The symbol Γ refers to the standard Gamma function, which is defined as $\Gamma(z)=\int_0^\infty t^{z-1}e^{-t}dt$. Using this definition and the third equation from system (1.1), we can directly derive (3.27). □

Lemma 3.6. *Let (u,v,z) be a solution to system (1.1) defined on $U\times(0,T_{max})$ with initial data (u_0,v_0,z_0) satisfying (1.3). Then, there exists a positive constant $C_{12}>0$, independent of t, such that*

$$\|u(\cdot,t)\|_{L^3(U)}\le C_{12}\quad \textit{for all } t\in(0,T_{max}). \tag{3.30}$$

Proof. By multiplying the initial equation of (1.1) by the variable u^2, we obtain

$$\begin{aligned}
&\frac{1}{3}\frac{d}{dt}\int_U u^3dx+2d_1\int_U u|\nabla u|^2dx+d\int_U u^4dx\\
&=2\chi\int_U u^2\nabla(G*z)\nabla udx+\int_U\Big(\frac{c\beta u^3v}{a+v}-\theta u^3\Big)dx\\
&\le d_1\int_U u\,|\nabla u|^2\,dx+\frac{2\chi^2}{d_1}\int_U u^3|\nabla(G*z)|^2dx+c\beta\int_U u^3dx\\
&\le d_1\int_U u\,|\nabla u|^2\,dx+\frac{2\chi^2}{d_1}\|u\|_{L^6}^3\|\nabla(G*z)\|_{L^4}^2+c\beta\|u\|_{L^3}^3\\
&\le d_1\int_U u\,|\nabla u|^2\,dx+\frac{2\chi^2G_0^2C_{11}^2}{d_1}\|u\|_{L^6}^3+c\beta\|u\|_{L^3}^3,
\end{aligned}$$

which implies that

$$\frac{d}{dt}\|u\|_{L^3}^3+\frac{4d_1}{3}\|\nabla u^{\frac{3}{2}}\|_{L^2}^2+3d\|u\|_{L^4}^4\le\frac{6\chi^2G_0^2C_{11}^2}{d_1}\|u\|_{L^6}^3+3c\beta\|u\|_{L^3}^3. \tag{3.31}$$

From (3.19), it follows that $\|u^{3/2}\|_{L^{4/3}}=\|u\|_{L^2}^{3/2}\le C_7^{3/2}$. To bound the term $\|u\|_{L^6}^3$ on the right-hand side of (3.31), we first note that $\|u\|_{L^6}^3=\|u^{3/2}\|_{L^4}^2$. We now apply a specific form of the two-dimensional Gagliardo-Nirenberg inequality in Lemma 2.5 to the function $\varphi=u^{3/2}$ in our two-dimensional domain ($n=2$). We choose the parameters $k=1$, $q=2$, $r=4/3$, and $p=4$. The exponent relation gives

$$\frac{1}{4}=\lambda\left(\frac{1}{2}-\frac{1}{2}\right)+\frac{1}{4/3}(1-\lambda)=\frac{3}{4}(1-\lambda),$$

which yields $\lambda=2/3$. Since $\lambda\in(0,1)$, the application is valid. The inequality thus becomes

$$\|u^{3/2}\|_{L^4}\le C_{GN}\left(\|\nabla u^{3/2}\|_{L^2}^{2/3}\|u^{3/2}\|_{L^{4/3}}^{1/3}+\|u^{3/2}\|_{L^{4/3}}\right).$$

Squaring both sides and using the fact that $(a+b)^2\le 2(a^2+b^2)$, we get

$$\|u^{3/2}\|_{L^4}^2\le 2C_{GN}^2\left(\|\nabla u^{3/2}\|_{L^2}^{4/3}\|u^{3/2}\|_{L^{4/3}}^{2/3}+\|u^{3/2}\|_{L^{4/3}}^2\right).$$

Using this result, we can estimate the term from (3.31)

$$\begin{aligned}\frac{6\chi^2G_0^2C_{11}^2}{d_1}\|u\|_{L^6}^3 &= \frac{6\chi^2G_0^2C_{11}^2}{d_1}\|u^{3/2}\|_{L^4}^2 \\ &\leq C_{13}\left(\|\nabla u^{3/2}\|_{L^2}^{4/3}\|u^{3/2}\|_{L^{4/3}}^{2/3} + \|u^{3/2}\|_{L^{4/3}}^2\right) \\ &\leq C_{13}'\left(\|\nabla u^{3/2}\|_{L^2}^{4/3}C_7 + C_7^2\right),\end{aligned} \tag{3.32}$$

where C_{13} and C_{13}' are positive constants. Now, we apply Young's inequality to the first term on the right-hand side. For any $\varepsilon > 0$

$$C_{13}'C_7\|\nabla u^{3/2}\|_{L^2}^{4/3} \leq \varepsilon\|\nabla u^{3/2}\|_{L^2}^2 + C_\varepsilon,$$

where C_ε is a constant depending on ε. Choosing $\varepsilon = \frac{4d_1}{3}$, we can absorb this term into the dissipation on the left-hand side of (3.31). This finally yields the estimate:

$$\frac{6\chi^2G_0^2C_{11}^2}{d_1}\|u\|_{L^6}^3 \leq \frac{4d_1}{3}\|\nabla u^{3/2}\|_{L^2}^2 + C_{14}, \tag{3.33}$$

where $C_{14} = \frac{9(\chi^2C_{11}^2)^3C_{13}^3C_7^3}{2d_1^3} + \frac{6\chi^2C_{11}^2}{d_1}C_{13}C_7^2$. Alternatively, by employing Young's inequality, we obtain

$$3c\beta\|u\|_{L^3}^3 \leq 3d\|u\|_{L^4}^4 + C_{15}, \tag{3.34}$$

where $C_{15} = \frac{c^4\beta^4}{4^4}|U|$. Then adding $\|u\|_{L^3}^3$ on both sides of (3.31) and substituting (3.32)-(3.34) into the results, one has

$$\frac{d}{dt}\|u\|_{L^3}^3 + \|u\|_{L^3}^3 \leq C_{16} + C_{17}.$$

This immediately yields (3.30) with $C_{12} = \left(\|u_0\|_{L^3}^3 + C_{16} + C_{17}\right)^{\frac{1}{3}}$. □

Equipped with improved spatial regularity for all solution components, we now complete the regularity bootstrap. Through semigroup estimates and parabolic smoothing, we finally achieve the critical L^∞ bound.

Lemma 3.7. *Let (u, v, z) be a solution to system (1.1) defined on $U \times (0, T_{\max})$. There exists a positive constant $C_{18} > 0$, independent of t, such that*

$$\|u(\cdot, t)\|_{L^\infty(U)} \leq C_{18} \quad \textit{for all } t \in (0, T_{\max}). \tag{3.35}$$

Proof. By Lemma 3.6 and (2.31), we have

$$\begin{aligned}\|\Phi(\cdot, t)\|_{L^3} =& \left\|rv\big(1 - \frac{v}{K}\big) - \frac{\beta uv}{a + v} + v\right\|_{L^3} \\ \leq& (M_1 + \frac{r}{K}M_1^2)|U|^{\frac{1}{3}} + \beta C_{12}.\end{aligned} \tag{3.36}$$

Applying ∇ to both sides of equation (3.29) and utilizing $Lp - Lq$ estimates, we may deduce from equation (3.36) that

$$\|\nabla v(\cdot, t)\|_{L^\infty} \leq \|\nabla e^{(d_2\Delta - 1)t}v_0\|_{L^\infty} + \int_0^t \|\nabla e^{(t-s)(d_2\Delta - 1)}\Phi(\cdot, s)\|_{L^\infty} ds$$

$$
\begin{aligned}
&\leq \xi_2\|v_0\|_{W^{1,\infty}} + \int_0^t \|\nabla e^{(t-s)d_2\Delta}\Phi(\cdot,s)\|_{L^\infty} ds \\
&\leq \xi_2\|v_0\|_{W^{1,\infty}} + \xi_2 \int_0^t \left((t-s)^{-\frac{5}{6}}+1\right) e^{-\lambda_1 d_1 (t-s)} \|\Phi(\cdot,s)\|_{L^3} ds \\
&\leq \xi_2\|v_0\|_{W^{1,\infty}} + \xi_2\Big((M_1+\frac{r}{K}M_1^2)|U|^{\frac{1}{3}}+\beta C_{12}\Big)\int_0^\infty (1+z^{-\frac{5}{6}})e^{-\lambda_1 d_2 \eta} d\eta \\
&\leq \xi_2\|v_0\|_{W^{1,\infty}} + \frac{\xi_2}{d_2\lambda_1}\Big((M_1+\frac{r}{K}M_1^2)|U|^{\frac{1}{3}}+\beta C_{12}\Big)\left(1+\Gamma(1/6)(d_2\lambda_1)^{\frac{5}{6}}\right),
\end{aligned}
$$

which gives

$$
\big\|\nabla v(\cdot,t)\big\|_{L^\infty} \leq C_{19}, \tag{3.37}
$$

where

$$
C_{19} = \xi_2\|v_0\|_{W^{1,\infty}} + \frac{\xi_2}{d_2\lambda_1}\left((M_1+\frac{r}{K}M_1^2)|U|^{\frac{1}{3}}+\beta C_{12}\right)\left(1+\Gamma(1/6)(d_2\lambda_1)^{\frac{5}{6}}\right).
$$

Then, from the third equation of (1.1), we can easily obtain that there exists a $C_{20} > 0$ such that

$$
\Big\|\nabla z(\cdot,t)\Big\|_{L^\infty} \leq C_{20}. \tag{3.38}
$$

We then rephrase the first equation of (1.1) in the following manner

$$
u_t - d_1\Delta u + u = -\chi\nabla\cdot(u\nabla(G*z)) + \frac{c\beta uv}{a+v} - \theta u - du^2 + u. \tag{3.39}
$$

By utilizing the formula of variation of constants on equation (3.39), one has

$$
\begin{aligned}
u(\cdot,t) =& e^{t(d_1\Delta-1)}u_0 - \chi\int_0^t e^{(t-s)(d_1\Delta-1)}\nabla\cdot(u\nabla(G*z))(\cdot,s)ds \\
&+ \int_0^t e^{(t-s)(d_1\Delta-1)}\left(\frac{c\beta uv}{a+v} - \theta u - du^2 + u\right)(\cdot,s)ds \\
\leq& e^{t(d_1\Delta-1)}u_0 - \chi\int_0^t e^{(t-s)(d_1\Delta-1)}\nabla\cdot(u\nabla(G*z))(\cdot,s)ds + \int_0^t e^{(t-s)(d_1\Delta-1)}(c\beta+1)u(\cdot,s)ds,
\end{aligned}
$$

which implies that

$$
\begin{aligned}
\big\|u(\cdot,t)\big\|_{L^\infty} \leq& \big\|u_0 e^{t(d_1\Delta-1)}\big\|_{L^\infty} + \chi\int_0^t \big\|e^{(t-s)(d_1\Delta-1)}\nabla\cdot(u\nabla(G*z))(\cdot,s)\big\|_{L^\infty} ds \\
&+ \int_0^t \big\|e^{(t-s)(d_1\Delta-1)}(c\beta+1)u(\cdot,s)\big\|_{L^\infty} ds.
\end{aligned} \tag{3.40}
$$

Using Lemma 2.4, and (3.30), one has

$$\left\|e^{(d_1\Delta-1)t}u_0\right\|_{L^\infty} \le 2\xi_1\left\|u_0\right\|_{L^\infty} \tag{3.41}$$

and

$$\begin{aligned}
&\int_0^t \left\|e^{(t-s)(d_1\Delta-1)}(c\beta+1)u(\cdot,s)\right\|_{L^\infty}ds\\
&\le (c\beta+1)\xi_1\int_0^t \left(1+(t-s)^{-\frac{1}{3}}\right)e^{-(t-s)}\left\|u(\cdot,s)\right\|_{L^3}ds\\
&\le (c\beta+1)\xi_1C_{12}\int_0^\infty (1+\eta^{-\frac{1}{3}})e^{-\eta}d\eta\\
&\le (c\beta+1)\xi_1C_{12}(1+\Gamma(\frac{2}{3})).
\end{aligned} \tag{3.42}$$

Alternatively, by utilizing the L^p-L^q estimate Lemma 2.4 and acknowledging that $C_0^\infty(U)$ is densely packed within $L^p(U)$ for all $1\le p<\infty$ and estimates (3.30) and (3.37), we have

$$\begin{aligned}
&\chi\int_0^t \left\|e^{(d_1\Delta-1)(t-s)}\nabla\cdot(u\nabla(G*z))(\cdot,s)\right\|_{L^\infty}ds\\
&\le \chi\xi_4\int_0^t (1+(t-s)^{-\frac{5}{6}})e^{-d_1\lambda_1 t}\left\|u(\cdot,s)\right\|_{L^3}\left\|\nabla(G*z)\right\|_{L^\infty}ds\\
&\le \chi\xi_4C_{12}C_{19}G_0\int_0^\infty (1+\eta^{-\frac{5}{6}})e^{-\lambda_1 d_1\eta}d\eta\\
&= \frac{\chi\xi_4C_{12}C_{19}G_0}{d_1\lambda_1}(1+\Gamma(1/6)(\lambda_1d_1)^{\frac{5}{6}}).
\end{aligned} \tag{3.43}$$

Substituting (3.41), (3.42)and (3.43), into (3.40), one has (3.35). □

By combining the above proofs, we can derive the global existence of the solution to (1.1), which is summarized in the following theorem.

Theorem 3.1. *Let U be a two-dimensional torus, representing a square with periodic boundary conditions. Assume that the initial data $(u_0(x),v_0(x),z_0(x))$ satisfy (1.3). Then, the system (1.1) admits a unique classical solution*

$$(u(x,t),v(x,t),z(x,t))\in\left[C^0\Big(U\times[0,\infty)\Big)\cap C^{2,1}\Big(U\times(0,\infty)\Big)\right]^3,$$

and there exists a constant $C>0$ such that

$$\|u(\cdot,t)\|_{L^\infty(U)}+\|v(\cdot,t)\|_{W^{1,\infty}(U)}+\|z(\cdot,t)\|_{W^{1,\infty}(U)}\le C \quad \text{for all } t\ge 0.$$

4. Global stability analysis

Clearly, system (1.1) has a boundary equilibrium $E_1 = (0, K, \frac{bK}{\gamma})$, and the positive steady states satisfy

$$\begin{cases} \frac{c\beta v}{a+v} - \theta - du = 0, \\ r\left(1 - \frac{v}{K}\right) - \frac{\beta u}{a+v} = 0, \\ bv - \gamma z = 0. \end{cases} \tag{4.1}$$

From the first equation of (4.1), we have $u = \frac{1}{d}\left(\frac{c\beta v}{a+v} - \theta\right)$, and substitute this into (4.1) yields

$$(dr)v^3 + (2adr - Kdr)v^2 + (dra^2 - 2Kdra + Kc\beta^2 - K\theta\beta)v - Kdra^2 - K\beta\theta a = 0. \tag{4.2}$$

Clearly, if $2a > K$, then (4.2) has a unique positive root denoting it by v^*. Furthermore, if $\frac{c\beta v^*}{a+v^*} - \theta > 0$, then system (1.1) has a unique positive steady state $E^* = (u^*, v^*, z^*)$, where

$$u^* = \frac{1}{d}\Big(\frac{c\beta v^*}{a + v^*} - \theta\Big), z^* = \frac{bu^*}{\gamma}.$$

In the subsequent analysis, we investigate the global stability of nonnegative equilibria by constructing a suitable Lyapunov functional. This approach is further supported by the application of Barbălat's lemma, as detailed in the following lemma.

Lemma 4.1 *(Barbălat's lemma [8]). Suppose that $f(t) : [1, \infty) \to \mathbb{R}$ is uniformly continuous and that $\lim\limits_{t\to\infty} \int_1^t f(\tau)d\tau$ exists, then $\lim\limits_{t\to\infty} f(t) = 0$.*

Furthermore, it is imperative that we ascertain a higher degree of regularity in the solutions as outlined below.

Lemma 4.2. *Consider the triplet (u, v, z), which is the only global, bounded classical solution of system (1.1) as shown in Theorem 3.1. For any $0 < s < 1$,*

$$\big\|u(\cdot,t)\big\|_{C^{2+s,1+\frac{s}{2}}(\bar{U}\times[1,\infty))} + \big\|z(\cdot,t)\big\|_{C^{2+s,1+\frac{s}{2}}(\bar{U}\times[1,\infty))} + \big\|v(\cdot,t)\big\|_{C^{2+s,1+\frac{s}{2}}(\bar{U}\times[1,\infty))} \le C,$$

where C is a constant.

Proof. Due to the boundedness of the solution components (u, v, z) established in Theorem 3.1 and the L^p estimate, we have

$$\big\|u(\cdot,t)\big\|_{W_p^{2,1}(\bar{U}\times[j+\frac{1}{4},j+3])} + \big\|v(\cdot,t)\big\|_{W_p^{2,1}(\bar{U}\times[j+\frac{1}{4},j+3])} + \big\|z(\cdot,t)\big\|_{W_p^{2,1}(\bar{U}\times[j+\frac{1}{4},j+3])} \le C_1, \tag{4.3}$$

for all $j \ge 0$, where $C_1 > 0$ is a constant independent of t.

Subsequently, the Sobolev embedding theorem, when applied with a sufficiently big value of p, yields

$$\big\|u\big\|_{C^{1+s,\frac{1}{2}+\frac{s}{2}}(\bar{U}\times[\frac{1}{4},\infty)} + \big\|v\big\|_{C^{1+s,\frac{1}{2}+\frac{s}{2}}(\bar{U}\times[\frac{1}{4},\infty)} + \big\|z\big\|_{C^{1+s,\frac{1}{2}+\frac{s}{2}}(\bar{U}\times[\frac{1}{4},\infty)} \le C_2. \tag{4.4}$$

By utilizing (4.4) and employing the Schauder estimate to the second and third equation of system (1.1), we derive

$$\big\|v\big\|_{C^{2+s,1+\frac{s}{2}}(\bar{U}\times[j+\frac{1}{3},j+3])}, \big\|z\big\|_{C^{2+s,1+\frac{s}{2}}(\bar{U}\times[j+\frac{1}{3},j+3])} \le C_3,\ j \ge 0. \tag{4.5}$$

Hence,

$$\|v\|_{C^{2+s,1+\frac{s}{2}}(\bar{U}\times[\frac{1}{3},\infty))}, \|z\|_{C^{2+s,1+\frac{s}{2}}(\bar{U}\times[\frac{1}{3},\infty))} \le C_4. \tag{4.6}$$

The equation u of system (1.1) can be reformulated as

$$\frac{\partial u}{\partial t} - d_1\Delta u - \chi\nabla u \cdot \nabla(G * z) = \Psi(x,t), \tag{4.7}$$

where

$$\Psi(x,t) = \chi u\Delta(G * z) + u\left(\frac{c\beta v}{a+v} - \theta - du\right).$$

By (4.5) and (4.6), we obtain

$$\|\Psi\|_{C^{s,\frac{s}{2}}(\bar{U}\times[j+\frac{1}{3},j+3])} + \|\chi\nabla(G * z)\|_{C^{s,\frac{s}{2}}(\bar{U}\times[j+\frac{1}{3},j+3])} \le C_5, j \ge 0.$$

Subsequently, by invoking the classical parabolic Schauder estimate to equation (4.7), we have

$$\|u\|_{C^{2+s,1+\frac{s}{2}}(\bar{U}\times[j+1,j+3])} \le C_6 \quad \text{for all} \quad j \ge 0,$$

which leads to

$$\|u\|_{C^{2+s,1+\frac{s}{2}}(\bar{U}\times[1,\infty))} \le C_7. \quad \square \tag{4.8}$$

Having established the regularity of solutions in Lemma 4.2, we now analyze their global stability.

Lemma 4.3. *Suppose that (u, v, z) is a nonnegative global classical solution of (1.1) emanating from initial data fulfilling (1.3), and (u^*, v^*, z^*) is the positive steady state of (1.1). Then, there exists a constant C such that*

$$\|\nabla z\|_{L^2}^2 \le C\left(\|u-u^*\|_{L^2}^2 + \|v-v^*\|_{L^2}^2 + \|z-z^*\|_{L^2}^2\right). \tag{4.9}$$

Proof. From the second equation in system (1.1), it follows that

$$\begin{aligned} v_t =& d_2\Delta v + rv - \frac{rv^2}{K} - \frac{\beta u}{a+v} \\ =& d_2\Delta v + rv - \frac{rv_2}{K} - \frac{\beta u}{a+v} - \left(rv^* - \frac{rv^{*2}}{K} - \frac{\beta u^* v^*}{a+v^*}\right) \\ =& d_2\Delta v + \left(r - \frac{r}{K}(v+v^*) - \frac{a\beta u^*}{(a+v)(a+v^*)}\right)\tilde{v} - \frac{a\beta v}{(a+v)(a+v^*)}\tilde{u}, \end{aligned}$$

where $\tilde{v} = v - v^*$ and $\tilde{u} = u - u^*$.

Then, we have

$$\tilde{v}_t = d_2\Delta\tilde{v} + \left(r - \frac{r}{K}(v+v^*) - \frac{a\beta u^*}{(a+v)(a+v^*)}\right)\tilde{v} - \frac{a\beta v}{(a+v)(a+v^*)}\tilde{u}. \tag{4.10}$$

Multiplying (4.10) by $-\Delta\tilde{v}$ and then integrating over U, we have

$$\frac{1}{2}\frac{d}{dt}\left\|\nabla\tilde{v}\right\|_{L^2}^2 = -d_2\left\|\Delta\tilde{v}\right\|_{L^2}^2 - \int\limits_U \left(r - \frac{r}{K}(v+v^*) - \frac{a\beta u^*}{(a+v)(a+v^*)}\right)\tilde{v}\Delta\tilde{v}dx + \int\limits_U \frac{a\beta v}{(a+v)(a+v^*)}\tilde{u}\Delta\tilde{v}dx. \tag{4.11}$$

From Theorem 3.1, and by Hölder inequality and Young inequality, we have

$$\begin{aligned}\frac{1}{2}\frac{d}{dt}\left\|\nabla\tilde{v}\right\|_{L^2}^2 + d_2\left\|\Delta\tilde{v}\right\|_{L^2}^2 \leq & C_1\left(\int\limits_U \tilde{v}\Delta\tilde{v}dx + \int\limits_U \tilde{u}\Delta\tilde{v}dx\right) \\ \leq & C_1\left(\left\|\tilde{v}\right\|_{L^2}\left\|\Delta\tilde{v}\right\|_{L^2} + \left\|\tilde{u}\right\|_{L^2}\left\|\Delta\tilde{v}\right\|_{L^2}\right) \\ \leq & C_1\left(\frac{C_1}{2d_2}\left\|\tilde{v}\right\|_{L^2}^2 + \frac{C_1}{2d_2}\left\|\tilde{u}\right\|_{L^2}^2 + \frac{d_2}{2C_1}\left\|\Delta\tilde{v}\right\|_{L^2}\right).\end{aligned}$$

Thus, by Poincaré inequality, we obtain

$$\frac{d}{dt}\left\|\nabla\tilde{v}\right\|_{L^2}^2 \leq - d_2\left\|\nabla\tilde{v}\right\|_{L^2}^2 + C_3\left(\left\|\tilde{v}\right\|_{L^2}^2 + \left\|\tilde{u}\right\|_{L^2}^2\right).$$

Then, by Grönwall's inequality, we have

$$\left\|\nabla v\right\|_{H^1} \leq C_4\left(\left\|v-v^*\right\|_{L^2} + \left\|u-u^*\right\|_{L^2}\right). \tag{4.12}$$

Taking the spatial gradient of the third equation of system (1.1), one has

$$(\nabla z)_t = b\nabla v - \gamma\nabla z. \tag{4.13}$$

Then, from Eq. (4.13), it follows that

$$\left\|\nabla z\right\|_{H^1} \leq C_5\left(\left\|\nabla v\right\|_{L^2} + \left\|z-z^*\right\|_{L^2}\right). \tag{4.14}$$

In two-dimensional space, the standard Gagliardo-Nirenberg inequality states

$$\left\|\nabla z\right\|_{L^2}^2 \leq C_6\left(\left\|z-z^*\right\|_{L^2}\left\|\Delta z\right\|_{L^2} + \left\|z-z^*\right\|_{L^2}^2\right). \tag{4.15}$$

Using elliptic regularity and the estimates from (4.14), we have

$$\left\|\Delta z\right\|_{L^2} \leq C_7\left(\left\|u-u^*\right\|_{L^2} + \left\|z-z^*\right\|_{L^2} + \left\|v-v^*\right\|_{L^2}\right). \tag{4.16}$$

Substituting (4.16) into (4.15), and applying Young's inequality yields

$$\begin{aligned}\left\|\nabla z\right\|_{L^2}^2 \leq & C_8\left(\left\|z-z^*\right\|_{L^2}\left(\left\|u-u^*\right\|_{L^2} + \left\|v-v^*\right\|_{L^2} + \left\|z-z^*\right\|_{L^2}\right) + \left\|z-z^*\right\|_{L^2}^2\right) \\ \leq & C\left(\left\|u-u^*\right\|_{L^2}^2 + \left\|z-z^*\right\|_{L^2} + \left\|v-v^*\right\|_{L^2}^2\right),\end{aligned}$$

which proves (4.9). □

The following theorem establishes the global stability of the positive steady state of the system.

Theorem 4.1. *Assume that the following conditions hold:*

$$\chi^2 < \frac{d_1 r\gamma}{G_0 b}\left(\frac{r}{K} - \frac{\beta u^*}{a(a+v^*)}\right), \tag{4.17}$$

$$0 < \frac{r}{K} - \frac{\beta u^*}{a(a+v^*)} < \frac{2d_2 ca}{M_1(a+v^*)}. \tag{4.18}$$

Then, the positive steady state $E^* = (u^*, v^*, z^*)$ is globally asymptotically stable. Furthermore, there exist $T_0 > 0$, and two positive constants $K_1 > 0$ and $\lambda_1 > 0$ such that for all $t > T_0$, the following exponential decay estimate holds:

$$\left\|u - v^*\right\|_{L^\infty} + \left\|v - v^*\right\|_{L^\infty} + \left\|z - z^*\right\|_{L^\infty} \le K_1 e^{-\lambda_1 t}. \tag{4.19}$$

Proof. Define the following Lyapunov functional:

$$V_1(t) = \int_U \left(u - u^* - u^* \ln \frac{u}{u^*}\right) dx + \alpha_1 \int_U \left(v - v^* - v^* \ln \frac{v}{v^*}\right) dx + \frac{\alpha_2}{2} \int_U (z - z^*)^2 \, dx + \frac{1}{4} \int_U |\nabla z|^2 dx,$$

where $\alpha_1 = \frac{ca}{a+v^*}$, $\alpha_2 = \frac{\gamma}{b}\left(\frac{r}{K} - \frac{\beta u^*}{a(a+v^*)}\right)$.

Let

$$I_1(t) = \int_U \left(u - u^* - u^* \ln \frac{u}{u^*}\right) dx, I_2(t) = \int_U \left(v - v^* - v^* \ln \frac{v}{v^*}\right) dx,$$

$$I_3(t) = \frac{\alpha_2}{2} \int_U (z - z^*)^2 \, dx + \frac{1}{4} \int_U |\nabla z|^2 dx.$$

Next, differentiating $V_1(t)$ with respect to t, we have

$$\begin{aligned}
&\frac{d}{dt} I_1(t) = \int_U (1 - \frac{u^*}{u}) u_t dx \\
&= -d_1 u^* \int_U \frac{|\nabla u|^2}{u^2} dx + \int_U (u - u^*)\left(\frac{c\beta_1 v}{a+v} - \theta - du\right) dx \\
&= -d_1 u^* \int_U \frac{|\nabla u|^2}{u^2} dx + \chi \int_U \frac{\nabla u}{u} \nabla (G * z) dx + \int_U (u - u^*)\left(\frac{ac\beta(v - v^*)}{(a+v)(a+v^*)} - d(u - u^*)\right) dx \\
&\le -d_1 u^* \int_U \frac{|\nabla u|^2}{u^2} dx + \frac{d_1 u^*}{2} \int_U \frac{|\nabla u|^2}{u^2} dx + \frac{\chi^2}{2d_1 u^*} \int_U |\nabla (G * z)|^2 dx - d \int_U (u - u^*)^2 dx \\
&\quad + \int_U \frac{ac\beta}{(a+v)(a+v^*)} (u - u^*)(v - v^*) dx \\
&\le -\frac{d_1 u^*}{2} \int_U \frac{|\nabla u|^2}{u^2} dx + \frac{\chi^2 G_0}{2d_1 u^*} \int_U |\nabla z|^2 dx - d \int_U (u - u^*)^2 dx + \int_U \frac{ac\beta}{(a+v)(a+v^*)} (u - u^*)(v - v^*) dx.
\end{aligned}$$

Similarly, from the second and third equations of (1.1), we have

$$\frac{d}{dt}I_2(t) = -d_2 v^* \int\limits_U \frac{|\nabla v|^2}{v^2}dx + \int\limits_U (v-v^*)\Big(r - \frac{rv}{K} - \frac{\beta u}{a+v}\Big)dx,$$

$$= -d_2 v^* \int\limits_U \frac{|\nabla V|^2}{v^2}dx + \int\limits_U (v-v^*)\Big(-\frac{rv}{K} + \frac{rv^*}{K} - \frac{\beta u}{a+v} + \frac{\beta u^*}{a+v^*}\Big)dx,$$

$$= -d_2 v^* \int\limits_U \frac{|\nabla V|^2}{v^2}dx - \frac{r}{K}\int\limits_U (v-v^*)^2 dx - \int\limits_U \frac{\beta(a+v^*)}{(a+v)(a+v^*)}(u-u^*)(v-v^*)dx$$

$$+ \frac{\beta u^*}{(a+v)(a+v^*)}(v-v^*)^2 dx$$

and

$$\frac{d}{dt}I_3(t) = \int\limits_U (z-z^*)\, z_t dx + \frac{1}{2}\frac{d}{dt}\int\limits_U |\nabla z|(\nabla z)_t dx$$

$$= \int\limits_U (z-z^*)(bv-\gamma z)dx + b\int\limits_U \nabla v \nabla z dx - \gamma \int\limits_U |\nabla z|^2 dx$$

$$= -\gamma \int\limits_U (z-z^*)^2 dx + b\int\limits_U (z-z^*)(v-v^*)dx - \gamma\int\limits_U |\nabla z|^2 dx + b\int\limits_U \frac{\nabla v}{v} v\nabla z dx$$

$$\leq -\frac{\gamma}{2}\int\limits_U (z-z^*)^2 dx + \frac{b}{2\gamma}\int\limits_U (v-v^*)^2 dx - \frac{\gamma}{2}\int\limits_U |\nabla z|^2 dx + \frac{b^2 M_1^2}{2\gamma}\int\limits_U \frac{|\nabla V|^2}{v^2}dx.$$

Therefore, we have

$$\frac{d}{dt}V_1(t) = \frac{d}{dt}I_1(t) + \alpha_1 \frac{d}{dt}I_2(t) + \alpha_2 \frac{d}{dt}I_3(t) = L_1 + L_2,$$

where

$$L_1 = -d\int\limits_U (u-u^*)^2 dx - \alpha_1 \int\limits_U \Big(\frac{r}{K} - \frac{\beta u^*}{(a+v^*)(a+v)}\Big)(v-v^*)^2 dx - \frac{\alpha_2\gamma}{2}\int\limits_U (z-z^*)^2 dx$$

$$\leq -d\int\limits_U (u-u^*)^2 dx - \alpha_1 \int\limits_U \Big(\frac{r}{K} - \frac{\beta u^*}{(a+v^*)a}\Big)(v-v^*)^2 dx - \frac{\alpha_2\gamma}{2}\int\limits_U (z-z^*)^2 dx$$

$$L_2 = -\frac{d_1 u^*}{2}\int\limits_U \frac{|\nabla u|^2}{u^2}dx - \Big(d_2\alpha_1 - \frac{b^2 M_1^2 \alpha_2}{2\gamma}\Big)\int\limits_U \frac{|\nabla v|^2}{v^2}dx - \Big(\frac{\alpha_2\gamma}{2} - \frac{\chi^2 G_0}{2d_1 u^*}\Big)\int\limits_U |\nabla z|^2 dx.$$

Then,

$$\frac{d}{dt}V_1(t) \leq -\int\limits_U Y_1 B_1 Y_1^T dx - \int\limits_U Y_2 B_2 Y_2^T dx,$$

where $Y_1 = [u-u^*, v-v^*, z-z^*]$, $Y_2 = \left[\frac{\nabla u}{u}, \frac{\nabla v}{v}, \nabla z\right]$, and

$$B_1 = \begin{pmatrix} d & 0 & 0 \\ 0 & \alpha_1\Big(\frac{r}{K} - \frac{\beta u^*}{(a+v^*)a}\Big) & 0 \\ 0 & 0 & \frac{\alpha_2\gamma}{2} \end{pmatrix}, B_2 = \begin{pmatrix} \frac{d_1 u^*}{2} & 0 & 0 \\ 0 & d_2\alpha_1 - \frac{b^2 M_1^2 \alpha_2}{2\gamma} & 0 \\ 0 & 0 & \frac{\alpha_2\gamma}{2} - \frac{\chi^2 G_0}{2d_1 u^*} \end{pmatrix},$$

where M_1 is defined in Lemma 2.3. Under the conditions (4.17) and (4.18), B_1 and B_2 are both positive definite. Furthermore, it is posited that there exists a positive constant $\varepsilon_1 > 0$ such that $Y_1 B_1 Y_1^T \geq \varepsilon_1 |Y_1|^2$.

Consequently, we have

$$\frac{d}{dt} V_1(t) \leq -\varepsilon_1 \mathcal{F}_1(t) \tag{4.20}$$

with

$$\mathcal{F}_1(t) = \int_U (u-u^*)^2 dx + \int_U (v-v^*)^2 dx + \int_U (z-z^*)^2 dx.$$

We establish the non-negativity of the Lyapunov functional $V_1(t)$ through convex analysis. Define the convex entropy function

$$\phi(y) := y - u^* \ln y \quad \text{for } y > 0,$$

whose derivatives satisfy

$$\phi'(y) = 1 - \frac{u^*}{y}, \quad \phi''(y) = \frac{u^*}{y^2} > 0.$$

Applying Taylor's theorem with Lagrange remainder at $y = u^*$, there exists $\eta = \xi u + (1-\xi)u^*$ for some $\xi \in (0,1)$, such that

$$\phi(u) - \phi(u^*) = \frac{\phi''(\eta)}{2}(u-u^*)^2 = \frac{u^*}{2\eta^2}(u-u^*)^2 \geq 0. \tag{4.21}$$

This establishes

$$I_1(t) = \int_U \left[u - u^* - u^* \ln\left(\frac{u}{u^*}\right)\right] dx \geq 0.$$

Analogously, $I_2(t) \geq 0$ can be established. Consequently, the composite Lyapunov functional therefore satisfies:

$$V_1(t) = I_1(t) + I_2(t) + I_3(t) \geq 0, \quad \forall t > 0.$$

Next, integrating (4.20) over $t \in (1, \infty)$ yields

$$\int_1^\infty \mathcal{F}_1(t)\, dt \leq \frac{1}{\varepsilon_1} < \infty. \tag{4.22}$$

The uniform continuity of $\mathcal{F}_1(t)$ on $[1, \infty)$ follows from Lemma 4.2. Combining (4.22) with Lemma 4.1, we deduce

$$\mathcal{F}_1(t) = \int_U (z-z^*)^2\, dx + \int_U (u-u^*)^2\, dx + \int_U (v-v^*)^2\, dx \to 0, \quad \text{as} \quad t \to \infty,$$

which gives

$$\lim_{t\to\infty} \left(\left\|u-u^*\right\|_{L^2} + \left\|z-z^*\right\|_{L^2} + \left\|v-v^*\right\|_{L^2}\right) = 0. \tag{4.23}$$

Thus,

$$\left\|u-u^*\right\|_{W^{1,\infty}}+\left\|z-z^*\right\|_{W^{1,\infty}}+\left\|v-v^*\right\|_{W^{1,\infty}}\le K_1 \quad \text{for all} \quad t>1. \tag{4.24}$$

By utilizing the Gagliardo-Nirenberg inequality and referencing equation (4.24), it follows that

$$\left\|u-u^*\right\|_{L^\infty}\le K_2\left\|u-u^*\right\|_{W^{1,\infty}}^{\frac{1}{2}}\left\|u-u^*\right\|_{L^2}^{\frac{1}{2}}\le K_2K_1^{\frac{1}{2}}\left\|u-u^*\right\|_{L^2}^{\frac{1}{2}}, \tag{4.25}$$

which together with (4.23) implies

$$\lim_{t\to\infty}\left\|u-u^*\right\|_{L^\infty}=0. \tag{4.26}$$

Similarly, we have

$$\lim_{t\to\infty}\left(\left\|v-v^*\right\|_{L^\infty}+\left\|z-z^*\right\|_{L^\infty}\right)=0. \tag{4.27}$$

Next, we will derive the decay rates of the L^∞-norm. Applying the L'Hôpital's rule derives that

$$\lim_{u\to u^*}\frac{u-u^*-u^*\ln\frac{u}{u^*}}{\left(u-u^*\right)^2}=\lim_{u\to u^*}\frac{1-\frac{u^*}{u}}{2\left(u-u^*\right)}=\lim_{u\to u^*}\frac{1}{2u}=\frac{1}{2u^*}.$$

Thus, there exists a constant $t_1>0$ such that

$$\frac{1}{4u^*}\int_U\left(u-u^*\right)^2dx\le\int_U\left(u-u^*-u^*\ln\frac{u}{u^*}\right)dx\le\frac{1}{u^*}\int_U\left(u-u^*\right)^2dx \tag{4.28}$$

for $t>t_1$.

Similarly, we can find a constant $t_2>0$ such that

$$\frac{1}{4v^*}\int_U\left(v-v^*\right)^2dx\le\int_U\left(v-v^*-v^*\ln\frac{v}{v^*}\right)dx\le\frac{1}{v^*}\int_U\left(v-v^*\right)^2dx \tag{4.29}$$

holds for all $t>t_3$, where $t_3=\max\{t_1,t_2\}$. By the definition of V_1 and $\mathcal{F}_1$, along with (4.28), (4.29) and Lemma 4.3, we obtain that there exist two positive constants θ_1 and θ_2 such that

$$\theta_1\mathcal{F}_1(t)\le V_1(t)\le\theta_2\mathcal{F}_1(t) \tag{4.30}$$

for some $t>t_3$. Then the combination of (4.30) and (4.20) gives

$$\frac{d}{dt}V_1(t)+\frac{\varepsilon_1}{\theta_2}V_1(t)\le 0 \quad \text{for all } t>t_3,$$

which means that for all $t>t_3$, there exists a constant $\eta_1>0$ such that

$$V_1(t)\le V_1(t_3)e^{-\frac{\varepsilon_1}{\theta_2}t}\le\eta_1e^{-\frac{\varepsilon_1}{\theta_2}t}. \tag{4.31}$$

From (4.30)-(4.31), it follows that

$$\mathcal{F}_1(t)\le\frac{1}{\theta_1}V(t)\le\frac{\eta_1}{\theta_1}e^{-\frac{\varepsilon_1}{\theta_2}t}, \quad t\ge t_3.$$

Combining this with the expression of $\mathcal{F}_1(t)$, we obtain

$$\|u-u^*\|_{L^2}^2+\|z-z^*\|_{L^2}^2+\|v-v^*\|_{L^2}^2 \le \frac{\eta_1}{\theta_1}e^{-\frac{\varepsilon_1}{\theta_2}t} \quad \text{for all } t \ge t_3. \tag{4.32}$$

Thus, by combining (4.25) and (4.32), and Sobolev embedding $H^2(U) \hookrightarrow L^\infty(U)$, we enhance the spatial regularity to derive the L^∞-decay:

$$\|u(\cdot,t)-u^*\|_{L^\infty} \le \theta_3 e^{-\eta_2 t} \quad \text{for all } t \ge t_3, \tag{4.33}$$

where $\eta_2 > 0$ and $\theta_3 > 0$ are constants.

Analogous arguments applied to the v- and z-components yield

$$\|v-v^*\|_{L^\infty}+\|z-z^*\|_{L^\infty} \le K_0 e^{-\lambda_1 t} \quad \text{for all } t \ge t_3. \tag{4.34}$$

Synthesizing (4.33) and (4.34), we obtain the global exponential stability result (4.19) in $L^\infty(U)$-norm, thereby completing the proof. □

Theorem 4.2. *Suppose that*

$$cK\beta < a\theta. \tag{4.35}$$

Then, the constant steady state $E_1 = (0, K, \frac{bK}{\gamma})$ is globally asymptotically stable. Furthermore, there exist $T_1 > 0$, and a positive constant $K_3 > 0$ such that

$$\|u\|_{L^\infty}+\|v-K\|_{L^\infty}+\|z-\frac{bK}{\gamma}\|_{L^\infty} \le \frac{K_3}{1+t} \quad \textit{for all} \quad t > T_1. \tag{4.36}$$

Proof. Define the following Lyapunov functional:

$$V_2(t)=\int_U u dx+\alpha_1\int_U \left(v-K-K\ln\frac{v}{K}\right)dx + c\int_U \left(z-\frac{bK}{\gamma}\right)^2 dx.$$

Next, differentiating $V_2(t)$ with respect to t, we have

$$\begin{aligned}
\frac{d}{dt}V_2(t) =& \int_U \left(\frac{c\beta_1 uv}{a+v}-\theta u - du^2\right)dx - cd_2K\int_U \frac{|\nabla v|^2}{v^2}dx + c\int_U (v-K)\left(r-\frac{rv}{K}-\frac{\beta u}{a+v}\right)dx \\
&+ b\int_U (z-\frac{bK}{\gamma})(v-K)dx - \gamma\int_U (z-\frac{bK}{\gamma})^2 dx, \\
=& -cd_2K\int_U \frac{|\nabla v|^2}{v^2}dx - d\int_U u^2 dx - \frac{cr}{K}\int_U (v-K)^2 dx + \int_U \left(\frac{cK\beta}{a+v}-\theta\right)u dx \\
&+ b\int_U (z-\frac{bK}{\gamma})(v-K)dx - \gamma\int_U (z-\frac{bK}{\gamma})^2 dx.
\end{aligned}$$

Consequently, there exists a constant $\varepsilon_2 > 0$ such that

$$\frac{d}{dt}V_2(t) \le -\varepsilon_2\mathcal{F}_2(t) \tag{4.37}$$

with

$$\mathcal{F}_2(t) = \int_U u^2 dx + \int_U (v-K)^2 dx + \int_U \left(z - \frac{bK}{\gamma}\right)^2 dx.$$

Following the approach in the proof of Theorem 4.1, we can identify $T_1 > 0$ satisfying

$$\frac{1}{4K}\int_U (v-K)^2\,dx \le \int_U \left(v - K - K\ln\frac{v}{K}\right) dx \le \frac{1}{K}\int_U (v-K)^2\,dx. \tag{4.38}$$

Thus,

$$\begin{aligned} V_2(t) \le & C_1\left(\int_U u dx + \int_U (v-K)^2 dx + \int_U \left(z - \frac{bK}{\gamma}\right)^2 dx\right) \\ \le & C_2\left(\left(\int_U u^2 dx\right)^{\frac{1}{2}} + \left(\int_U (v-K)^2 dx\right)^{\frac{1}{2}} + \left(\int_U \left(z - \frac{bK}{\gamma}\right)^2 dx\right)^{\frac{1}{2}}\right) \\ \le & C_3 \mathcal{F}_2^{\frac{1}{2}}(t), \end{aligned}$$

which, combining with (4.37), we have

$$\frac{d}{dt}V_2(t) + \frac{\varepsilon_2}{C_3^2}V_2^2(t) \le 0. \tag{4.39}$$

Solving this ordinary differential inequality (4.39), we arrive at

$$V_2(t) \le \frac{C_4}{1+t} \quad \text{for all} \quad t \ge T_1.$$

Using the same argument as in the proof of Theorem 4.1, we readily get (4.36) and complete the proof. □

5. Numerical simulations

This section explores the role of the detection scale R in generating spatiotemporal patterns through nonlocal resource perception, using the top-hat detection function as a case study. The following parameters are employed:

$$\begin{aligned} &r = 0.9, \quad K = 4, \quad \beta = 0.8, \quad a = 1, \quad c = 0.9, \quad \theta = 0.2, \\ &d = 0.05, \quad b = 0.5, \quad \gamma = 0.6, \quad d_1 = 0.1, \quad d_2 = 0.1, \quad \chi = 0.1. \end{aligned} \tag{5.1}$$

5.1. Simulations in one-dimensional space

We consider the top-hat detection function is

$$G(x-y) := \begin{cases} \frac{1}{2R}, & \|x-y\| \le R, \\ 0, & \text{otherwise}, \end{cases} \tag{5.2}$$

and choose $U = [-5, 5]$. We implement finite difference methods and solve the model with MATLAB software.

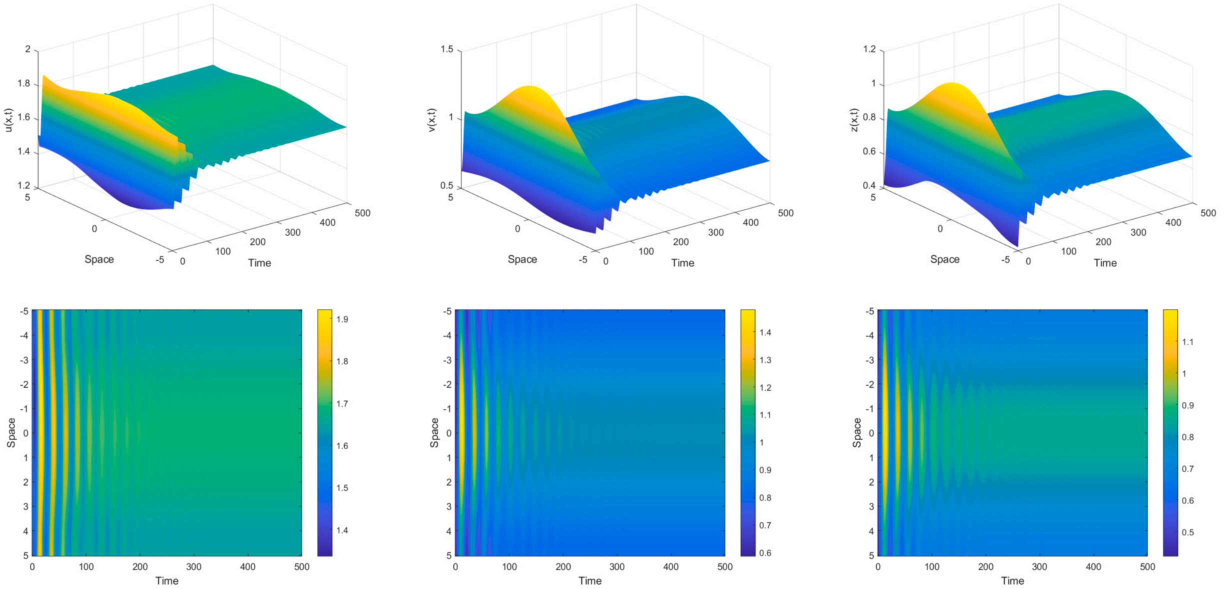

Fig. 1. The spatial pattern of system (1.1) with $R = 4$ and other parameters are in (5.1), and the initial conditions $u_0(x) = 0.6258 + 0.03\cos(\frac{\pi x}{5})$, $v_0(x) = 1.5429 + 0.05\sin(\frac{\pi x}{5})$ and $z_0(x) = 0.5215 + 0.1\cos(\frac{\pi x}{5})$. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

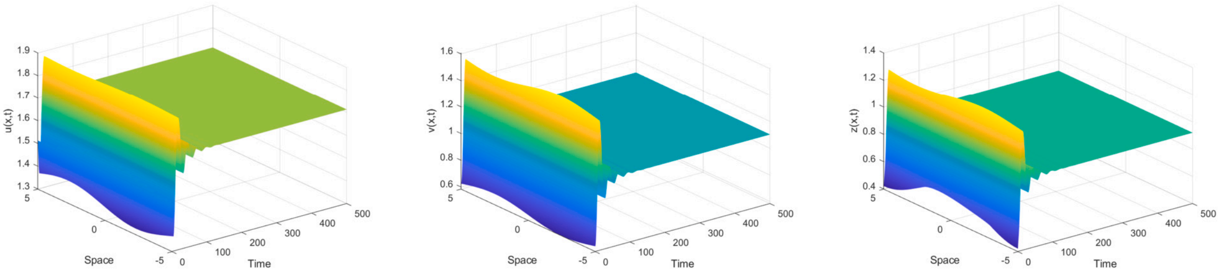

Fig. 2. The spatial pattern of system (1.1) with $R = 0.05$ and other parameters are in (5.1), and the initial conditions $u_0(x) = 0.6258 + 0.03\cos(\frac{\pi x}{5})$, $v_0(x) = 1.5429 + 0.05\sin(\frac{\pi x}{5})$ and $z_0(x) = 0.5215 + 0.1\cos(\frac{\pi x}{5})$.

Initially, for a relatively larger memory usage range, such as $R = 4$, the system exhibits distinct spatiotemporal patterns (see Fig. 1). When the memory usage range is set to $R = 0.05$, spatial patterning disappears; the system stabilizes into a homogeneous state (see Fig. 2). This indicates that restricted perception at small spatial scales localizes consumer movement and resource exploitation. Consequently, large-scale spatial heterogeneity diminishes, and both consumer and resource populations distribute uniformly across the domain. Such behavior aligns with the hypothesis that perceptual limitations impede consumers' capacity to detect and respond to resource gradients, resulting in uniform population distributions.

5.2. Simulations in two-dimensional space

Complementing the 1D results, we extend our analysis to two-dimensional space to examine pattern formation in planar ecological systems. We consider the top-hat detection function is

$$G(x-y) := \begin{cases} \frac{1}{\pi R^2}, & \|x-y\| \le R, \\ 0, & \text{otherwise}, \end{cases} \tag{5.3}$$

We choose $U = [-50, 50] \times [-50, 50]$.

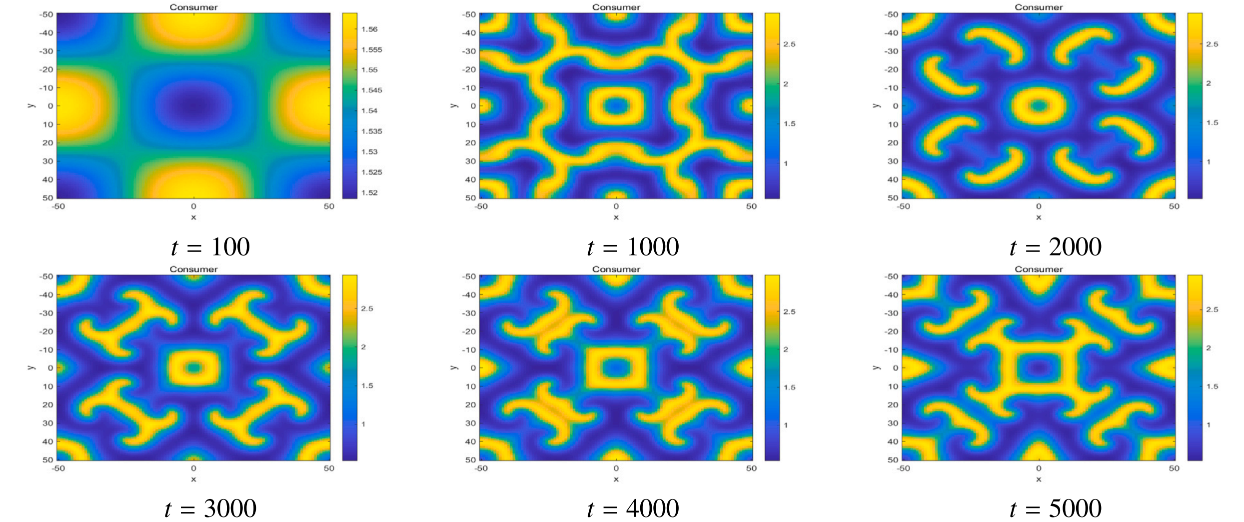

Fig. 3. The spatial pattern of system (1.1) with $L = 100$ and $R = 0.5$ and other parameters are in (5.1), and the initial conditions $u_0(x, y) = 0.6258 + 0.01\cos(\frac{\pi x}{50})\cos(\frac{\pi y}{50})$, $v_0(x, y) = 1.5429 + 0.01\cos(\frac{\pi x}{50})\cos(\frac{\pi y}{50})$, $z_0(x, y) = 0.5215 + 0.01\cos(\frac{\pi x}{50})\cos(\frac{\pi y}{50})$.

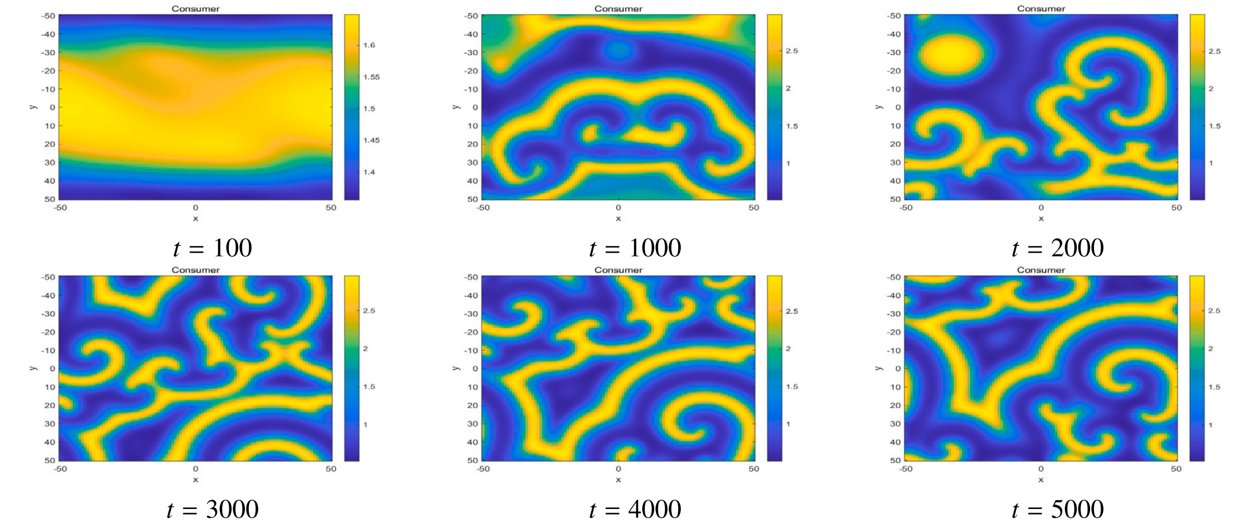

Fig. 4. The spatial pattern of system (1.1) with $L = 100$ and $R = 25$ and other parameters are in (5.1), and the initial conditions $u_0(x, y) = 0.6258 + 0.01\cos(\frac{\pi x}{50})\cos(\frac{\pi y}{50})$, $v_0(x, y) = 1.5429 + 0.01\cos(\frac{\pi x}{50})\cos(\frac{\pi y}{50})$, $z_0(x, y) = 0.5215 + 0.01\cos(\frac{\pi x}{50})\cos(\frac{\pi y}{50})$.

From Fig. 3 we find that when memory usage range $R = 0.5$ system exhibits regular spatial patterns. These patterns arise due to the balance between the consumers' ability to detect resources over a moderate spatial scale and the diffusion of both consumers and resources. The regular patterns indicate that consumers efficiently track and utilize resource patches, leading to the formation of stable, periodic structures in the spatial distribution of populations. Such patterns are often observed in ecological systems where species exhibit localized aggregation in response to resource availability.

Building on these observations, Fig. 4 reveals significantly different dynamics at maximal memory usage range ($R = 25$). The irregular spatial patterns emerging under extended detection capabilities suggest that a broader memory usage range allows consumers to detect and respond to resource gradients over larger distances, leading to more complex and dynamic interactions between consumers and resources. The irregular patterns may arise from the interplay between long-range perception, resource depletion, and consumer movement, yielding stochastic spatial distributions with reduced predictability and more dynamic. This phenomenon mirrors the complexity observed in natural ecosystems, where trophic interactions frequently produce dynamic spatial architectures through emergent self-organization.

6. Conclusion

This study establishes a cognitive consumer-resource framework incorporating nonlocal memory utilization and dynamic cognitive mapping through partial differential equations (PDEs). Our analysis demonstrates two fundamental mathematical properties of the system: global existence of classical solutions under periodic boundary conditions in two-dimensional space, and global stability of homogeneous steady states under specific parameter constraints, verified through Lyapunov functional analysis.

The model's innovative nonlocal perception mechanism, implemented via spatial convolution kernels, reveals crucial insights into consumer-resource dynamics. Numerical simulations highlight the critical role of memory usage range R in shaping ecological patterns and processes: Restricted perception (small R) confines consumers to local resource exploitation, resulting in homogeneous spatial distributions Extended perception (large R) enables complex pattern formation through nonlocal interactions, potentially enhancing resource exploitation efficiency through coordinated movement strategies.

These spatial self-organization phenomena provide mathematical evidence supporting ecological theories of biodiversity maintenance. Regular patterns may facilitate multispecies coexistence through distinct resource partitioning, while irregular configurations could reflect dynamic equilibria that promote ecosystem resilience. Our findings particularly emphasize how memory-driven cognitive processes can stabilize ecological systems by mediating consumer-resource interactions across spatial scales.

Declaration of competing interest

The authors declare that they have no conflict of interest.

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Data availability

No data were used in this study.

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