GAME THEORY

Rock Paper Scissors

Player 1 & Player 2

Rock > Scissors > Paper > Rock

Payoff Matrix for ← player 2 column

Player 1

$$A = \begin{array}{c} \\ R \\ P \\ S \end{array} \begin{array}{c} \begin{array}{ccc} R & P & S \end{array} \\ \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} \end{array}$$

↑ player 1 row

payoff for player 2

$$B = \begin{array}{c} \\ R \\ P \\ S \end{array} \begin{array}{c} \begin{array}{ccc} R & P & S \end{array} \\ \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} \end{array} = -A = A^T$$

player 2 ← (R P S)

player 1 ←

pure strategy

R P S

strategy vector: $\underline{x} \in \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right.$

$\left. \right\}$

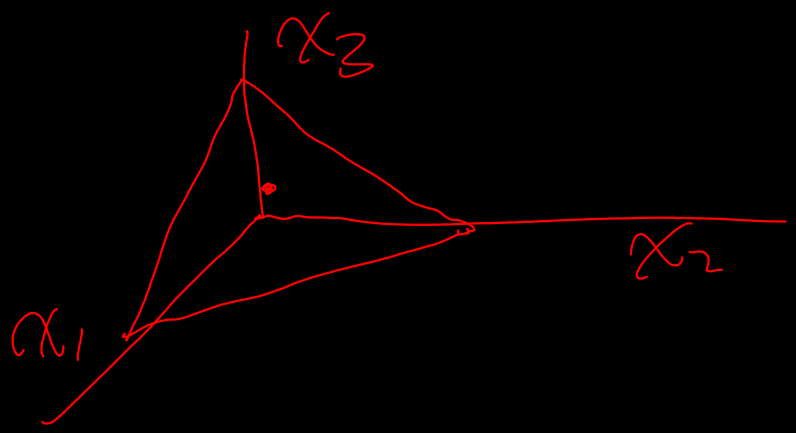
2: $\underline{y} \in$ //

Mixed strategy

$x = (x_1, x_2, x_3)$ $\quad x_1 + x_2 + x_3 = 1$

e.g. $x = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$

$$\underline{x} \in \Delta_3 = \{ x \in \mathbb{R}^3 : x_1 + x_2 + x_3 = 1, \; x_1, x_2, x_3 \geq 0 \}$$



Player 1: R S R S S R P R R R P

Player 2: R S R P S R R S P R

R P S

$\left(\frac{3}{5}, \frac{1}{5}, \frac{1}{5}\right)$ 1: R = 6, P = 2, S = 2

$\left(\frac{1}{2}, \frac{1}{5}, \frac{3}{10}\right)$ 2: R = 5, P = 2, S = 3

mixed strategies

Prisoner's Dilemna's

2 thieves A&B arrested

	C	D
C	1	3
D	0	2

confess

→ A & B confess: 2 year each

only A confesses: 3 years for B

only B confesses: 3 years for A

cooperation: neither confess / stay silent: 1 year each

individual rationality does not lead to collective rationality.

Tragedy of the Commons

- over grazing on common land
- over fishing

Battle of the sexes

Man \ Woman	soccer	ballet
soccer	3	0
ballet	-1	1

Woman =

1	-1
0	3

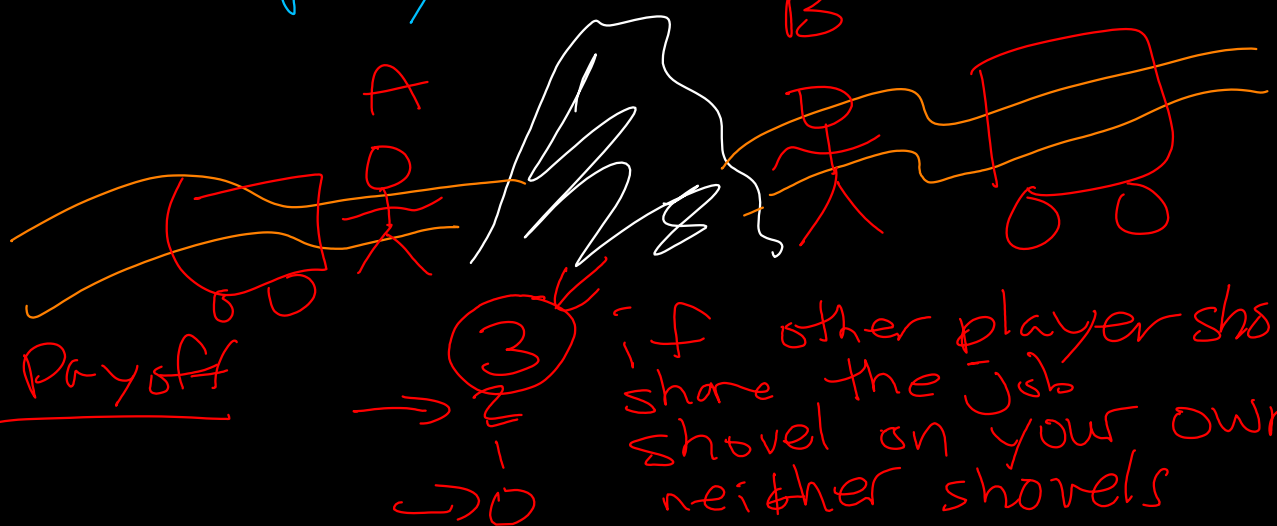
man

best payoff is when both agents co-operate.

Co-ordinating Game

Snowdrift problem

Anti-coordinate ; best to pick opposite strategy of other player



C = cooperate

D = defect (don't cooperate partner)

	C	D
C	$r-\frac{c}{2}=2$	$r-c=1$
D	$r=3$	0

$r>0$

$0<c<r$

$r=3$

$c=2$

Classification of 2 X 2 games

Pay off matrix $A = \begin{matrix} & C & D \\ C & R & S \\ D & T & P \end{matrix}$

$R, S, T, P \in R$

1) Prisoner's Dilemma:
$T > R > S > P$

2) Coordination Game
$R > T \quad P > S$

3) Snowdrift Game:-
$T > R \quad S > P$