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Reckoning and Reasoning

or

The Joy of Rote

by

by Klaus Hoechsmann†

You might have heard of this story, but it bears being repeated. In 1992, Lou D’Amore, a science teacher in the Toronto area, sprung a Grade 3 arithmetic test from 1932 on his Grade 9 class, and found that only 25% of his students could do all of the following questions.

1. Subtract these numbers: 9,864 – 5,947
2. Multiply: 92 × 34
3. Add the following: $126.30 + $265.12 + $196.40
4. An airplane travels 360 kilometers in three hours. How far does it go in one hour?
5. If a pie is cut into sixths, how many pieces would there be?
6. William bought six oranges at 5 cents each and had 15 cents left over. How much had he at first?
7. Jane had $2.75. Mary had 95 cents more than Jane. How much did Jane and Mary have together?
8. A boy bought a bicycle for $21.50. He sold it for $23.75. Did he gain or lose and by how much?
9. Mary’s mother bought a hat for $2.85. What was her change from $5?
10. There are 36 children in one room and 33 in the other room in Tom’s school. How much will it cost to buy a crayon at 7 cents each for each child?

This modest quiz quickly rose to fame as “The D’Amore Test.” Other teachers tried it on their classes, with similar results. There was some improvement in Grades 10 to 12, where 27% of students could get through it, but they tend to be keener anyway since their less ambitious classmates usually give up on quantitative science after Grade 9. All in all, the chance of acing the D’Amore Test appears to be independent of anything learned in high school.

At first glance this seems as it should be, because the test certainly contains no “high school material”. On second thought, however, a strange asymmetry appears: while all students expect to use the first two R’s (Readin’ and Ritin’) throughout their schooling and beyond, they drop the third R (Rithmetic) as soon as they can—if indeed they acquired it at all. Has it always been like this? I doubt it: my grandmother went to school only twice a week (being needed in yard and kitchen) but was later able to handle all the arithmetic in her little grocery store without prior attendance of remedial classes. She did not even have a cash register.

To many administrators, think-tankers, etc., this is beside the point, because we now live in the brave new computer age. A highly placed person who has likely never repaired a car engine, and probably knows little about computers, suggested that 20 years ago, “an auto mechanic needed to be good at working with his hands,” whereas now he needs Algebra 11 and 12 to run his array of robots. For a more insights of this kind, you might wish to visit www.geocities.com/Eureka/Plaza/2631/articles.html, where electricians, machinists, tool-and-die makers, and plumbers are also included “among those who need Grade XI or XII algebra.” It doesn’t say what for.

Mechanics laugh at this: remember the breaker-point gaps, ignition timing, engine compression, battery charge, alternator voltage, headlight angle, and a multitude of other numerical values we had to juggle in our minds and check with fairly simple tools—today’s gadgets make our jobs more routine, they say. But ministerial bureaucrats tend to believe the hype, with a fervour proportional to their distance from “Mathematics 12,” which has gobbled up Algebra 12 in most places I know.

Aye, there’s the rub: the third R has morphed into the notorious M. “What’s in a name?”, you ask, “that which we called rithmetic by any other word would sound as meek.” How many times must you be told that M is hard and boring, and hear the refrain “I have never been good at M”? It is the perfect cop-out, acceptable even in the most exclusive company—a kind of egalitarian salute by which “normal” members of the species homo sapiens recognize one another. How can a teacher of, say, social studies be expected to develop vivid lessons around unemployment, national debt, or global warming—as long as these topics are mired in M? He/she still must mention numbers, to be sure, but can now present them in good conscience as disconnected facts, knowing that his/her students’ minds will be uplifted in another class, by that lofty but (to him/her) impenetrable M.

Ask any marketing expert: labels are not value-free, they attract, repel, or leave you indifferent. Above all, they raise expectations, which, in the case of M, are as manifold and varied as the subject itself. Is it conceptualization, exploration, visualization, constructivism, higher-order thinking, problem solving—or all of the above? The guessing and experimenting goes on and on, producing bumper crops of learned papers and theses, conferences, surveys, and committees, as well as confused students and teachers. “This is the first time in history that Jewish children cannot learn arithmetic” said an

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Israeli colleague, referring to the state of Western style education in his country, where the recent Russian immigrants maintain a parallel school system.

Not every country has followed the R to M conversion. In the Netherlands and (what was) Yugoslavia, children still learn rekenen and račun, respectively, together with reading and writing. The more weighty M is left for later. Germany clung to Rechnen till the 1960’s, and then rashly followed the American lead, pushing Mathematik all the way down to Kindergarten—with the effect of finding itself cheek-to-jowl with the US (near the end of the list) in international comparisons.

I hear the sound of daggers being honed: what is this guy trying to sell (in this culture we are all vendors), is it “Back to Basics”? Does he hanker for “Drill and Kill,” for “Top Down” at a time when all good men and women aspire to “Bottom Up”? Readers unaccustomed to Educators’ discourse might be puzzled at such extreme positions getting serious attention. They would immediately see middle ground between tyranny and anarchy, boot camp and nature trail, etc. Why do we always argue Black versus White? I really cannot explain it. Maybe it is because we need strident voices and must hold single notes as long as we can, in order to be noticed in this mighty chorus. How did we get here?

Although the benefits of planned obsolescence are obvious, they are not often mentioned to justify the present trend toward immumeracy. It is the relentless advance of technology which must be seen as the main reason for the retreat of archaic skills. Speech-recognizing computers already exist, and once they are mass-produced, writing will not need to be taught anymore, at least not at public expense. Whatever we now do with our hands and various other body-parts outside the brain will clearly fall into the domain of sports. Only in this spirit does it make sense to climb a mountain top that can be more safely reached by helicopter.

Before the advent of electric and later electronic calculators, computations had to follow rigid algorithms that allowed the boss or auditor to check them. This was “procedural knowledge” of an almost military kind—justly despised and rejected when it became obsolete. Oddly enough it did, however, have an important by-product: by sheer habit, simple calculations were done at lightning speed, and often mentally—of course with a large subconscious component. In many places, this “mental arithmetic” was even practised as a kind of sport, still “procedural,” in some sense, but open to improvisation—more like soccer than like target shooting.

Look at the first question of the D’Amore Test: 9, 864 − 5, 947. Abe did it the conventional way and had to “borrow” twice. Beth zeroed in on the last three digits, noting that 947 exceeded 864 by 36 + 47 = 83, which she subtracted from 4000. Chris topped up the second number by 53 to 6000 and hence had to increase the first one to 9, 864 + 53 = 9, 917. Dan and Edith had yet different ways, but all got 3, 917. On the second question, Abe again used the standard method, since he was a bit lazy but meticulous. Beth looked at the 92 and thought 100 − 10 + 2, playing it very safe. Chris spotted one of his favourite short-cuts: 3 × 17 = 51, and reasoned that 9 × 34 = 6 × 51 = 306, and so on. Dan was attracted to the fact that 92 was twice 46, which lies as far above 40 as 34 lies below it. Therefore 46 × 34 was 1600 − 36, which had to be doubled to 3200 − 72. Edith blurted out the answer 3128 and said she did not remember how she got it.

When I was in Grade 7, I knew such kids—and was irked by the fact that many played this mental game as well as they played soccer. Justice was restored when, in Grade 8, they were left in the dust by x and y but continued to outrun me on the playing field. Maybe they never missed the x and y in later life (unlike contemporary plumbers), but I am almost sure their “number sense” often came in handy. Today’s kids are to acquire this virtue by doing brain-teasers and learning to “think like mathematicians,” carefully avoiding “mindless rote.”

Whenever I walk by the open door of a mathematician’s workplace, I see black or white boards covered with calculations and diagrams. How come they get to indulge in this “rote,” while kids must fiddle with manipulations or puzzle till their heads ache? Could it be that we mathematicians sometimes engage in “mindful rote”—the kind known to musicians and athletes? If so, we ought to step out of the closet and tell the world about the joy of rote. Anyone who has observed young children will immediately know what we mean.

And while we’re at it, we might reclaim ownership of the M-word, at least suggest that it be kept out of the K-4 world. This does not mean that schools should go back to teaching ‘rithmetic—admittedly an awkward label. How about “reckoning and reasoning,” a third and fourth R to balance the first two? They would be associated with good old common sense, and, as Descartes has pointed out, nobody ever complains of not having enough of that.

There are 10 kinds of mathematicians. Those who can think in binary and those who can’t…

Two math professors are hanging out in a bar.

“You know,” the first one complains. “Teaching mathematics nowadays is pearls for swine: the general public is completely clueless about what mathematics actually is.”

“You’re right!” says his colleague. “Look at the waitress. I’m sure she has no clue about any math she doesn’t need to give out correct change—and maybe not even that.”

“Well, let’s have some fun and put her to the test,” the first prof replies. He waves the waitress to their table and asks: “Excuse us, but you seem to be an intelligent young woman. Can you tell us what the square of a+b is?”

The girl smiles: “That’s easy: it’s a² + b²…”

The professors look at each another with a barely hidden smirk on their faces, when the waitress adds: “…provided that the field under consideration has characteristic two.”

Q: What is the difference between a Ph.D. in mathematics and a large pizza?

A: A large pizza can feed a family of four…

A French mathematician’s pick up line: “Vouslez-vous Cauchy avec moi?”