Leonhard Euler

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This year, St. Petersburg celebrates its 300th anniversary. This gorgeous city, one of the most beautiful in the world, was created by the desire and power of a single man—Russian czar Peter the Great. Peter founded the city in 1703 in an empty and swampy place. He sought to transform Russia into a more civilized, cultured, and developed country, to “westernize” it. Peter was changing Russia, almost rebuilding, and for this he needed a new face, a new capital, the most beautiful and luxurious city in Europe. The construction of St. Petersburg was extremely difficult. Thousands of workers died from disease, the cold, and the unbearable living conditions. But Peter had no time to wait. In 1712, the capital of the Russian empire moved from Moscow to St. Petersburg. Peter wanted not only to build the city of his dreams, he wanted St. Petersburg to be the cultural and scientific center of Russia. There was a lack of educated, skilled people in Russia at the time—it’s hard to believe, but not all Russian noblemen could read and write, and very few spoke foreign languages. Peter sent the youth of Russian nobility to study abroad, and invited foreign specialists in different fields to work in Russia. Many architects, sculptors, and engineers from Denmark, Holland, France, Germany, Italy, and other countries came to Russia, building and decorating its cities, creating its navy, forming its industry. When Peter died, his wife Catherine the First continued his reforms. In 1725, she established the Russian Academy of Science. As was common in those times, she invited many foreigners to work in the newly-created Academy. Many great scientists came to the young capital of the Russian Empire. Among them was one of the leading mathematicians of the 18th century—Leonhard Euler.

Euler was born in Basel, Switzerland on April 15, 1707. His father was a pastor and, as was usual, he wanted his son to also become a minister. He sent Euler to the University of Basel to study theology. However, it turned out that the young man had a gift for mathematics and loved it very much. Johann Bernoulli, the Swiss mathematician, paid attention to the talented student and convinced the elder Euler to allow his son to change his specialization to mathematics. Euler continued to study at the University of Basel and finished in 1726. He published his first research paper in 1726 and his second in 1727. His work on the best arrangement of masts on a ship was submitted for the Grand Prize of the Paris Academy of Science and won second place. That was a big achievement for the young mathematician. In 1726, Euler was offered the physiology post at the Russian Academy of Science.

The Russian Academy of Science in Saint Petersburg.

He accepted the offer and arrived in St. Petersburg in 1727. D. Bernoulli and J. Hermann, who were already working at the Russian Academy, helped Euler to join the mathematics–physics division, which meant that he also became a full member of the Academy. The same year, Euler married Katherine Gzel, daughter of a Swiss painter who worked in St. Petersburg.

In 1736, Euler published the two-volume work “Mechanica, sive motus scientia analytice exposita,” where he applied
mathematical analysis methods to the problems of motion in a vacuum and in a resisting environment. This work earned him world fame. Euler developed some of the first analytical methods for the exact sciences; he started to apply differentiation and integration to physical problems. By 1740, Euler had attained a very high profile, having won the Grand Prize of the Paris Academy of Science in both 1738 and 1740. He had also written the wonderful “Direction to Arithmetic,” which was later translated into Russian. It was the first Russian book to represent arithmetic as a mathematical science.

Frederick the Great

In 1740, after the death of the Empress Anna Ioanovna, two-month-old Ioan IV was declared Emperor of Russia. As he was too young to rule, his mother, Anna Leopoldovna, became regent. Living in Russia became dangerous, especially for foreigners, and Euler decided to accept the invitation of Frederick the Great, the King of Prussia, to work in Berlin. There, Euler was met with great respect and was given the freedom to pursue his research as he wished. However Euler didn’t completely end his work for Russian Academy. He was still partially paid by Russia, and he continued to write reports for the Academy and teach young Russians who arrived in Berlin. The Russians respected him so much that when his house was destroyed by Russian troops during the Russian–Prussian war, Euler received full compensation.

Euler’s 25 years in Berlin were very busy and productive. He enjoyed great mathematical success and also found time to accomplish all kinds of social work. For example, he served on the Library and Scientific Publications Committee of the Berlin Academy and was a government advisor on state lotteries, insurance, annuities and pensions, and artillery.

Euler wrote nearly 380 articles during his Berlin period. He also wrote many scientific and popular science books, including his famous “Letters to a Princess of Germany,” which was translated into many languages and published almost 40 times. He also led the Berlin Academy of Science after the death of Maupertuis in 1759, although he never held the formal title of President.

Euler’s phenomenal ability to work is demonstrated by the fact he produced about 800 pages of text per year. That would be a significant number even for a novelist; for a mathematician, it is hardly believable. Euler made a big contribution to analysis, geometry, trigonometry, and number theory, and introduced such notation as $f(x)$ for function, $\sum$ for sum, $e$ for the base of natural logarithm, $\pi$ for the ratio of the length of a circle to its diameter, and $i$ for imaginary unit. Euler proved the following formula for a convex polyhedron: $V + F = 2 + E$, where $V$ is number of vertexes of the polyhedron, $F$ is number of faces of the polyhedron, and $E$ is number of edges of the polyhedron. This formula has the extension, very important in topology, called Euler characteristics. In addition to his work in mathematics, Euler published works in philosophy, astronomy, physics, and mechanics.

Using the graph theory that he introduced, Euler solved the following famous problem, the so-called “Königsberg’s Bridges Problem.”

**Problem:** The Pregel river in Königsberg has the form shown in the picture below. There are seven bridges across it. Would it be possible, walking through the town, to cross each bridge exactly once?

Königsberg Bridges

Euler was able to show that this is impossible; moreover he described precisely the form of the river and bridges required to reach an affirmative solution.

Catherine the Great

In 1762, the politics in Russia changed again. Empress Catherine II, later named “Catherine the Great,” came to the throne. The atmosphere in Russian society improved dramatically. Catherine was an extraordinary person, very talented and educated. She aimed to create in Russia a regime of “Educated Absolutism.” She invited many progressive people to Russia—she was in correspondence with Voltaire, she wrote books and plays, and she was very interested in art and in science. Catherine II also started one of the most famous and beautiful museums in the world, the Hermitage. The time of her rule is called the “Gold Century” of Russian history. Catherine the Great understood very well that the country couldn’t prosper without science. She knew also that the science could enhance her prestige. She increased the budget of the Academy to 60 000 rubles per year, which was much more than the budget of the Berlin Academy.

Catherine II offered Euler an important post in the mathematics department, conference-secretary of the Academy, with a big salary. She instructed her representative in Berlin to agree to his terms if he didn’t like her first offer, to ensure that he would arrive in St. Petersburg as soon as possible.

In 1766, Euler returned to St. Petersburg. Soon after, he became almost blind due to a cataract in his left eye (his right eye was already very poor). However, that didn’t stop him from working. Euler dictated his works to a young boy, who wrote them in German. In 1771, his home was destroyed by
fire and he was able to save only himself and all of his mathematical manuscripts except the “New Theory of the Motion of the Moon.” Fortunately, Euler had an exceptional memory, which helped him restore the manuscript quite quickly. After the fire, Euler was obliged to move into a new house, the interior of which was unknown to him. This was extremely difficult for a blind old man.

In September 1771, Euler had surgery to remove his cataract. The surgery took only three minutes and was very successful—the mathematician’s vision was restored. Doctors advised Euler to avoid bright light and overloading his eyes; reading and writing were forbidden. Unfortunately, Euler didn’t take care of his eyes; he continued to work and after a few days lost his vision again, this time without any hope of recovery. Euler took this quietly, with great courage. Amazingly, his productivity only increased. Despite his total blindness, Euler wrote almost half of his articles after his return to St. Petersburg.

In 1773, Euler’s wife died. They were together almost 40 years and had 13 children. At that time, the mortality rate for children was very high, and eight of their children died before reaching adulthood. After his wife’s death, Euler continued to work diligently, using his son’s and some of his student’s eyes for reading. He worked until September 18, 1783, the last day of his life. According to his biographer, on that day Euler gave a mathematics lesson, worked on mathematics, and discussed with Lexell and Fuss the planet Uranus, recently discovered by astronomers. He died in the evening.

References:
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Denis Diderot was a French philosopher in the 18th century. He traveled Europe extensively, and on his travels stopped at the Russian court in St. Petersburg. His wit and suave charm soon drew a large following among the younger nobles at the court—and so did his atheist philosophy. That worried Empress Catherine the Great very much...

Swiss mathematician Leonhard Euler was working at the Russian court at that time and, unlike Diderot, he was a devout Christian. So, the Empress asked him for help in dealing with the threat posed by Diderot.

Euler had himself introduced to Diderot as a man who had found a mathematical proof for the existence of God. With a stern face, the mathematician confronted the philosopher: “Monsieur, \((a + b^n)/n = x\) holds! Hence, God exists. What is your answer to that?”

The usually quick-witted Diderot was speechless. Laughed at by his followers, he soon returned to France.