On the Dynamics of Karate

by Florin Diacu[†]

The origins of martial arts can be traced back to ancient times and the systems of self-defense and fighting designed by Oriental priests and Asian warriors. Derived from those systems, karate, meaning "empty hand," was developed in Okinawa in the early 17th century after the Japanese conquered the island and banned the use of all weapons. Today, millions of people are practising karate all over the world. There exist many karate styles, four of which are recognized by the World Karate Federation: goju, shito, shotokan, and wado. No style is superior to any other. All of them lead to similar results, but each of them follows specific ideas and reflects a different philosophy.

Picture 1

Sally Chaster, 1st dan black-belt and chief instructor at the Kuwakai club in Victoria, executes a forward punch (junzuki).

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Our goal here is to use a few simple mathematics and physics tools to analyze the dynamics of karate and to draw several conclusions on the efficiency of various techniques. Let us start with taking a look at the forward punch (junzuki, see Picture 1). When performing a junzuki, the goal of the karate practitioner (karateka) is to keep the body in balance and achieve maximum energy when the knuckles hit the target. The fist travels a straight distance and rotates by approximately 180 degrees. Assuming that the rotation is uniform and that the fist and the forearm are approximated with a cylinder of radius r, the energy E is given by the formula:

$$E = mgh + \frac{1}{2}mv^2 + \frac{1}{2}mr^2\omega^2,$$

where m is the mass used in the punch, h is the difference in the height of the body from the initial position to the position when the punch hits the target (when stepping forward, the body drops 15 to 20 cm), $g = 9.8$ m/s^2 represents the gravitational acceleration, v denotes the velocity of the fist, and ω is the angular velocity of the fist's rotation. The three terms appearing on the right side of this equation are called: potential, kinetic, and rotational energy, respectively.

This formula allows us to draw several conclusions.

1. The greater the mass, the higher the energy. We see that the energy grows linearly with the mass. This implies that if X is twice as heavy as Y, then X converts two times more energy than Y. Apparently, we cannot do much about this quantity, which depends on the frame of the body. An arm usually weighs about 10 percent of a person's total body, but we can increase the mass of a punch by stepping forward. However, unlike street fighters, who often engage most of their body mass in a punch at the expense of losing their balance, the karateka chooses to use less mass to favour stability. As we will see below, there are better ways to increase the energy of a punch or a kick without losing balance and becoming vulnerable to a counterattack.

2. The lower the drop, the higher the energy. The above formula shows that the energy grows linearly with the difference in height, h, when the body is dropped. The potential energy, mgh, is a substantial source since it uses the entire mass of the body. The importance of this component will become clear in the numerical example given in remark 4.

3. The higher the speed, the higher the energy. Unlike mass and height difference, which are linear quantities, speed influences energy quadratically. This means that if X and Y have the same mass but X is twice as fast as Y, then X will produce four times more energy; if X is three times faster than Y, X will produce nine times more energy. This shows that speed is an essential component in karate and in any other physical fighting game. Fast punches and kicks are not important only because they surprise the opponent, but also because of their efficiency in producing energy. A karateka who breaks boards and bricks manages to achieve the highest speed at the moment of impact.

To better appreciate the importance of speed, let us note that some simple computations show the following facts:

- If X weighs 50 kg and Y weighs 70 kg, then X must punch only 18 percent faster to achieve the same effect as Y;
- If X weighs 50 kg and Y weighs 100 kg, then X must punch only 41 percent faster to achieve the same effect as Y.

This shows that women, who are in general smaller than men, can deliver equally effective punches if they increase their speed.

4. The effect of the fist's rotation is negligible. Contrary to what most people think, the effect of the fist's rotation is negligible in a punch. The best way to see this is through a numerical example.

Suppose that a karateka weighing 70 kg performs a forward punch (junzuki). Assume that the mass involved in the punch is that of the arm alone (approximately 7 kg). The average speed achieved by a black-belt karateka's fist at the moment of impact is about 7 m/s (see the table below) and that of the first rotation, ω, is about 5π rad/s (i.e., the fist rotates 180 degrees in 0.2 seconds). Let us also assume that the radius r of the cylinder that approximates the fist and the forearm is 3 cm = 0.03 m. The drop in height is approximately 20 cm = 0.2 m. Then the potential energy, E_P, kinetic energy, E_K, and rotational energy, E_R, take the following values, measured in Joule (J) (recall that 1 J = 0.239 calories):

$$E_P = mgh = 70 \times 9.8 \times 0.2 = 137.2 \text{ J},$$

$$E_K = \frac{1}{2}mv^2 = \frac{1}{2} \times 7 \times 7^2 = 171.5 \text{ J},$$

$$E_R = m(r\omega)^2 = \frac{1}{2} \times 7 \times (0.03 \times 5\pi)^2 = 0.78 \text{ J}.$$

This shows that the rotation accounts for 0.45% of the kinetic energy, 0.57% of the potential energy, and only 0.25% of the total energy. One quarter of a percentage point is a negligible quantity. However, we can see that the energies converted by the drop in height and the motion of the arm are comparable. This also explains the principle of keeping the body at the same height in order to conserve energy. Every up-and-down move by only 20 cm uses almost as much energy as a punch at 7 m/s.

5. The longer the distance, the higher the energy. We will now show that the energy changes linearly with the distance the fist travels from the time of initiating the punch to the time of impact. For this, recall the following two physics formulas:

$$v = at \quad \text{and} \quad L = \frac{1}{2}at^2,$$

which indicate that the velocity, v, equals the acceleration, a, times the time, t, and that the length, L, equals half the acceleration times the square of the time. Eliminating t from the two formulas, we obtain

$$v = \sqrt{2La},$$

which means that the speed increases with the square root of the distance. Substituting v into the expression of the kinetic energy, it follows that

$$E_K = mLa.$$

This proves the linear dependence of the energy on the distance, and shows that a longer arm can reach a higher speed at impact. However, there is a drawback to this, which we will discuss next.

6. The longer the distance, the longer the time. Intuitively, this should be clear to anybody. However, the linear dependence is not. Eliminating the acceleration from the two formulas written before, we obtain

$$t = \frac{2L}{v}.$$

This means that a shorter arm will reach the target linearly faster. In other words, if X's fist travels half the distance of Y's, Y will generate two times more kinetic energy but will need double the time to reach the target. In practice, this is not entirely true since v is not constant. The speed versus the position looks like the curve in Figure 1, as it is experimentally shown in [1]. However, the linear dependence between time and length is valid.

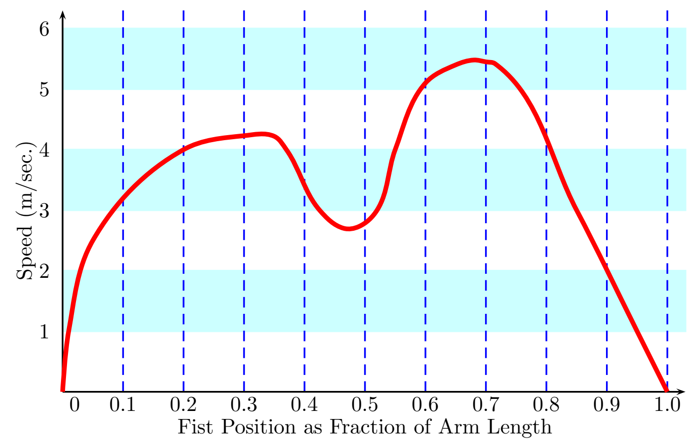


Figure 1

The speed of a fist in a forward punch as a function of its position. Data taken from a high-speed movie by J.D. Walker of Cleveland State University.

Except for the result about rotation in remark 4, these conclusions are also true in the case of kicks (e.g., the front kick, or maegeri, see Picture 2). Kicks, however, are more efficient than punches, not only because of the greater mass of the leg, which can reach up to 20 percent of a person's body, but especially due to the higher speed of the kick.

A comparative experimental study for the speeds of different techniques was done in [2]. The following table summarizes the conclusions obtained by the authors.

Technique	Max. speed
Front forward punch (junzuki)	5.7 – 9.8 m/s
Downward hammerfist block (otoshiuke)	10 – 14 m/s
Downward knife hand strike (shutouke)	10 – 14 m/s
Front kick (maegeri)	9.9 – 14.4 m/s
Side kick (yokogeri)	9.9 – 14.4 m/s
Roundhouse kick (mawashigeri)	9.5 – 11 m/s
Back kick (ushirogeri)	10.6 – 12 m/s

Table 1.

Speeds of different techniques.

We can now draw the following conclusion:

7. Kicks are between three and six times more powerful than punches. In the example described in remark 4, we computed the average potential, kinetic, and rotational energy of a junzuki punch performed by a black-belt karateka weighing 70 kg. We found the total energy was 309.48 J. Assuming now that the leg of the same person weighs 14 kg, that the speed of the kick is 12 m/s, and that there is no drop in height when executing the kick, we obtain that the energy developed by the technique, given by the kinetic energy alone, is

$$E = E_K = \frac{1}{2}mv^2 = \frac{1}{2} \times 14 \times 12^2 = 1008 \text{ J}.$$

This shows that the front kick is at least three times stronger than the forward punch. If the punch is executed without stepping forward and dropping the body, then its energy is 171.5 J, which is almost six times less than the value obtained for the kick.

Picture 2

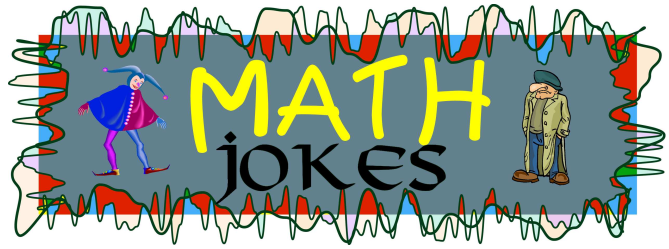
Norma Foster, 6th dan black-belt and the highest-ranked wado-karate woman in the world, performing a front kick (maegeri).

Similar estimates can be done for all kicks and punches, verifying the conclusion stated at the beginning of remark 7.

References

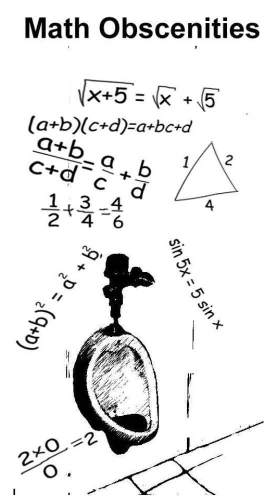
[1] Walker, J.D.: Karate Strikes, American Journal of Physics, **43** (1975), 845–849.

[2] Wilk, S.R., McNair, R.E., and Feld, M.S.: The Physics of Karate, American Journal of Physics, **51** (1983), 783–790.



A math professor, a native Texan, was asked by one of his students: "What is mathematics good for?"

He replied: "This question makes me sick! If you show someone the Grand Canyon for the first time, and he asks you, 'What's it good for?' what would you do? Well, you'd kick that guy off the cliff!"



A mathematician organizes a raffle in which the prize is an infinite amount of money paid over an infinite amount of time. Of course, with the promise of such a prize, his tickets sell like hot cakes.

When the winning ticket is drawn and the jubilant winner comes to claim his prize, the mathematician explains the mode of payment: "1 dollar now, 1/2 dollar next week, 1/3 dollar the week after that ..."

Q: What do you get if you cross an elephant with a grape?

A: $|\text{elephant}||\text{grape}| \cdot \sin(\theta)$.

Theorem. Every positive integer is interesting.

Proof. Assume that this is not so; that is, there are uninteresting positive integers. Then there must be a smallest uninteresting positive integer. But being the smallest such number is an extremely interesting property!