Part I: Introduction

Tic-Tetris-Toe is very much like Tic-Tac-Toe. The classic game is played on a 3 by 3 board, taking a square in each turn. Whoever is first to get 3 squares in a row or 3 on a diagonal wins. However, in this new game, we make two changes.

First, while we still play on a square board, it does not have to be 3 by 3. Tic-Tetris-Toe is actually five different games, each with a board of a different size. Second, we try to get different shapes. We use those from the popular video game Tetris, as shown in Figure 1.

There are actually seven different pieces, but since they are allowed to turn over, we have only five Tic-Tetris-Toe games. We call these pieces N4, L4, T4, I4 and O4, because they each have four squares and look like the letters N, L, T, I and O, respectively.

For N4, we play on a 3 by 3 board. For L4, we play on a 4 by 4 board. For T4, we play on a 5 by 5 board. For I4, we play on a 7 by 7 board. For O4, we play on a 9 by 9 board. Of course, the advantage is with the first player. Can you figure out a way for a sure win if I let you go first? Try these games with your friends, and then check below. Don’t peek—that will spoil the fun!

Part II: The N4 Game

Let us label the rows of the board 1, 2, and 3, and the columns a, b, and c. That way, each square will have a name. For example, the square at the bottom left corner will be called a1.

You will need at least four moves to win, and you will have an extra fifth move that may come in handy in some scenarios. Mapping out a winning strategy requires that you look quite far ahead. On the other hand, I (the opposing player) may be able to stop you from winning with one or two moves. Perhaps you should first consider what my strategy will be.

Figure 2 shows three ways in which I can stop you from winning. In each case, even if I give you all of the remaining squares, you still cannot complete N4. This tells you that you must take b2 in your first move, and make sure that you take at least one of b1 and b3, and at least one of a2 and c2.

Note that once you have taken b2, you do not have to worry about me sneaking up on you for a surprise win. You will do no worse than a draw. It would be too embarrassing to lose as the first player. Can I still stop you from winning? After you have taken b2, I really have two different choices: taking an edge square or a corner one.

Suppose I take a2. You already know that you must take c2. Now I give up. In your third move, you can take b1 and create a double-threat at a3 and c1. If I prevent you from doing this by taking one of these three squares, you will take b3 and create a double-threat at a1 and c3. Figure 3 shows that the key to your success is the W5 shape.

Am I better off if I start with a corner square, say a1? Suppose you still take c2. After all, it has worked once. Now I know that I must take one of a3, b3, and c1. You can force me to take c3 on my next move by taking b1 yourself. Then you can create a double-threat by taking one of c3, a2, and a3, depending on my move.

Can you remember all of this? You do not have to do that. Just understand that you must have b2, one of a2 and c2, and one of b1 and b3. Then look for double threats. With a little bit of practice, you will always win if you move first.
Part III: The L4, T4 and I4 Games

Label the extra rows 4, 5 and so on, and the extra columns d, e and so on. You can have an easy win in the L4 game. Start by taking b2. You are guaranteed to get b3 or c2 in your next move. If both are still there and both b1 and b4 are still empty, take b3. Otherwise, a2 and d2 will be empty, so take c2. On your third move, complete 3 squares in a row, and I cannot stop you from completing L4 on your fourth move. Since I have only made three moves so far, I cannot beat you to it.

You can win the T4 game by starting at the obvious place, c3. I can make one of five essentially different responses, at a1, a2, a3, b2, or b3. On your second move, you take d4. On your third move, you take either c4 to create a double-threat at b4 and c5, or d3 to create a double-threat at d2 and e3. I can neither stop you nor beat you to it.

The I4 game is the only one of the five that can be played competitively. While you have a sure win, it cannot be forced until your eighth move. In trying for the win, it is possible that you may set up a double-threat for me. Figure 5 shows a sample game in which I put up a good fight. On the seventh move, you either take c6 for the double-threat at a6 and e6, or take b4 for the double-threat at b3 and b7. I cannot stop both.

Part IV: The O4 Game

This game holds a big surprise. Even though the board looks more than large enough, you will not be able to force a win. I have a very simple but effective counter strategy that will prevent you from winning, even if we play on an infinite board. It is an elegant idea which demonstrates the beauty of mathematics.

I will combine pairs of adjacent squares into dominoes in the pattern of a brick wall, into which you will bash your head in vain. Whenever you take a square, I will take the other square of the same domino. As shown in Figure 6, no matter how you fit in O4, it must contain a complete domino. Since you can only have half of it, you cannot win!

Part V: Further Projects

Problem 1.
Find four connected shapes of three squares or less, joined edge-to-edge.

Remark: These shapes are called the monomino O1, the domino I2, and the trominoes I3 and V3. None of them provides much challenge as a game—the first player has an easy win if the board is big enough. This is because each of these pieces form parts of other pieces for which the first player can win. Our Tetris pieces are the tetrominoes. If we go to the pentominoes, you will find these games much more challenging. There are twelve such pieces, called F5, I5, L5, N5, P5, T5, U5, V5, W5, X5, Y5, and Z5, as shown in Figure 7. Pentomino is a registered trademark of Solomon Golomb, who has written a wonderful book called Polyominoes. This word means, “shaped or formed of many squares.” After the pentominoes come the 35 hexominoes, 108 heptominoes, 369 octominoes, and so on.

Problem 2.
Since P5 contains O4, and the domino strategy of Figure 6 works for the O4 game, the second player can also force a draw in the P5 game, even if it is played on an infinite board. On the other hand, there are four pentominoes that do not contain O4, but for which the domino strategy of Figure 6 also works. Which pentominoes are they?

Problem 3.
Show how the first player can force a win for the N5 game on a 6 by 6 board, and for the L5 and Y5 games on a 7 by 7 board.

Problem 4.
Match each of the other four pentominoes with one of the patterns in Figure 8 for a domino strategy.
Problem 5.
How many of the hexominoes contain a pentomino for which the second player has a domino strategy?

Problem 6.
Find domino strategies for the second player in games using the hexominoes in Figure 9. For one of them, you will have to find a new pattern.

Problem 7.
With the possible exception of the hexomino in Figure 10, no polyominoes formed of six or more squares offer the first player a sure win, even on an infinite board. Can a win be forced in a game using this hexomino?

Problem 8.
Returning to Tic-Tetris-Toe, it is easy to see that there is no win for the first player in the N4 game if it is played on a 2 by 2 board, because it is not even big enough to hold the piece. The L3 game on a 3 by 3 board is also a draw, if played properly. The classic Tic-Tack-Toe is still a draw even with additional winning configurations. Can the first player still force a win in the T4 games on a 4 by 4 board, or the I4 game on a 6 by 6 board?

Part VI: Acknowledgement

This article is based on Martin Gardner’s Mathematical Games column in Scientific American magazine, April, 1979. It has since been collected into the anthology Fractal Music, Hypercards and More, as Chapter 13, under the title Generalized Tic-tac-toe. This book was published by W. H. Freeman and Company, New York, in 1992. The original work was done by the noted graph theorist Frank Harary.

A mother of three is pregnant with her fourth child. One evening, her eldest daughter says to her dad, “Do you know, daddy, what I’ve found out?”

“No.”

“The new baby will be Chinese!”

“What?!”

“Yes. I’ve read in the paper that statistics show that every fourth child born nowadays is Chinese….”

A father who is very much concerned about his son’s poor grades in math decides to register him at a religious school. After his first term there, the son brings home his report card; he gets ‘A’s in math.

The father is, of course, pleased, but wants to know, “Why are your math grades suddenly so good?”

“You know,” the son explains, “when I walked into the classroom the first day and saw that guy nailed to a plus sign on the wall, I knew one thing—this place means business!”

“What happened to your girlfriend, that really smart math student?”

“She is no longer my girlfriend. I caught her cheating on me.”

“I don’t believe that she cheated on you!”

“Well, a couple of nights ago I called her on the phone, and she told me that she was in bed wrestling with three unknowns….”

Q: Why do mathematicians often confuse Christmas and Halloween?

Q: How do you make one burn?
A: Differentiate a log fire.

* Write to us if you get this joke!