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SPEED WINDSURFING: MODELING AND NUMERICS

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This paper is dedicated to Professor Max D. Gunzburger on the occasion of his 60th birthday.

Abstract. We consider the following problem: Given a finite number of sails of different sizes with prescribed shapes, given the speed of the wind, the weight of a surfer, the size and shape of a surfboard and a corresponding fin; determine the sail and the course such that the windsurfer achieves maximal speed. We develop a simple model for describing the movement of the windsurfer for maximizing the speed in terms of a nonlinear ODE (ordinary differential equation). We conclude with the presentation of corresponding numerical results, some remarks on their validation and more involved models.

Key Words. Speed windsurfing, simple model, flow of air particles, nonlinear ODE, numerical simulation.

1. Introduction

A common goal for many windsurfers is to achieve enough speed for *planing*, a certain state of gliding over the water surface, provided that the waves are not too high. This state is, loosely speaking, comparable to throwing a flat stone over the water surface such that it hits the surface several times before sinking. Naturally, planing depends on the speed of the surfer which in turn depends on the size of the board and its fin and of the size of the sail.

In the following, we always assume that the surfer knows to surf with an optimal technique to reach the maximal speed and that his/her weight uniquely determines the strength with which he/she is able to hold the sail by means of the boom with both the hands and/or a harness, a device by which the full body weight is used to provide an additional 'anti'-force to the force generated by the wind in the sail and the fin in the water. Moreover, any experienced windsurfer knows which board to pick to balance between stability and speed: the larger the board, the stabler it is but also the slower, and vice versa.

It should be mentioned that the current world record of 48.7 kn (90.2 km/h) in the 10 sqm class has been achieved by Finian Maynard with a board of size 220 cm (64 ltr) with a 28 cm fin and a 5.0 m^2 sail; the weight of the surfer was 117 kg [8, 11]. This world record was achieved on April 10th, 2005, in the 'French Trench' near Les Stes. Maries de la Mer (France) where a certain artificial tunnel generates very high wind speeds and which is too narrow for the sea water to produce any waves. Thus, ideally, the maximal speed seems to be achievable with a large sail and high wind speed but a very flat water surface, which is a condition that hardly occurs in practice.

The present paper is concerned with modeling the movement of the windsurfer and with determining over a finite number of given sails of prescribed shape the

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optimal sail by which the windsurfer can reach maximal velocity. Naively, one would think that the maximal speed will be caused by picking the sail with largest area of attack for the wind. However, the surfer needs to be able to hold the sail with hands and body weight, using a harness. That is, too large a sail for the weight and strength of the surfer might force him to fall into the water. In that situation, picking too large a sail may present an additional problem: in case of a sea level too deep for him to stand, he would have to use a water start which is very energy-consuming and, already after a few unsuccessful attempts, a dangerous enterprise, in particular, in case of offshore wind with high wind speed and large waves, which, in turn, often do not allow for a beginner-type basic start.

In summary, it is of vital importance for the practical surfer to pick the sail of the right size which allows him both for planing and, at the same time, is safe enough for him to use even in case of offshore wind. It is the purpose of the present paper to model and simulate this problem. Simultaneously, we wish to provide a program which runs on a common laptop computer at a beach station in an amount of time which is less than five minutes, an estimated time that an experienced surfer needs to switch the sail twice.

This paper is structured as follows. In Section 2, we introduce a basic model for the physical effects appearing in windsurfing and derive a nonlinear algebraic equation which gives the maximal speed of the windsurfer. Its approximate solution is described in Section 3, using Newton's second law and an explicit Runge-Kutta solver for the resulting nonlinear ODE. The numerical results are validated and discussed in Section 4, together with an outlook how to improve the basic model.

2. Modeling

Following the development of a first basic model in [9], we begin with notation and some physical assumptions.

We always assume that for some sea the speed of the wind $\overrightarrow{W_o}$, a board and a surfer with perfect technique are given.

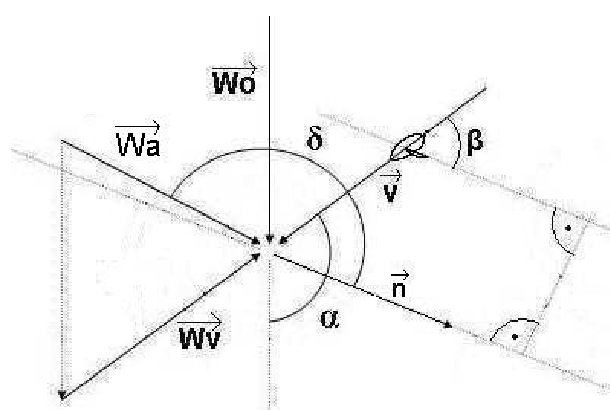

FIGURE 1. Movement of the surfer with local coordinate system.

The situation for modeling the movement of the surfer on the board is as follows. The surfer moves in direction $\overrightarrow{v}$ with angle α from the wind in downwind direction and holds the sail with angle β from the board, see Figure 1. Denote by $\overrightarrow{n}$ the vector for the position of the sail. Thus, α is the angle between $\overrightarrow{W_o}$ and $-\overrightarrow{v}$ and β the angle between $-\overrightarrow{v}$ and $\overrightarrow{n}$. All angles refer to the local coordinate system. The wind in the local coordinate system moving with the surfer, the so-called apparent wind $\overrightarrow{W_a}$, is composed of the original wind $\overrightarrow{W_o}$ and the fair wind denoted by $\overrightarrow{W_v}$ as

$$\overrightarrow{W_a} = \overrightarrow{W_o} + \overrightarrow{W_v} = \overrightarrow{W_o} - \overrightarrow{v} \tag{1}$$

employing $\overrightarrow{W_v} = -\overrightarrow{v}$; for the vectorial treatment of physical quantities, see, e.g., [3], Kap. 11. Finally, let δ be the angle between the apparent wind and the sail, that is, between $\overrightarrow{W_a}$ and $\overrightarrow{n}$.

We assume in our simple model that the air stream around the sail is laminar and that the air is incompressible and of vanishing viscosity, which is a usual assumption in view of the employment of Bernoulli's equation in (11) below.

Any experienced surfer will keep the sail during this movement perpendicular to the surface of the water. We furthermore assume that the friction resistance of the board can be completely described in terms of a known drag coefficient c_w. In this model, we have neglected factors like leeway and dependencies on weights. It will suffice in the following to consider a two-dimensional representation of the model since all employed velocities and forces lie in a plane parallel to the water surface.

We begin by relating the speed of the surfer to given quantities. His maximal achievable speed is characterized by the fact that he is not subjected to further acceleration. This means that the sum of all forces affecting the surfer and the board must vanish. We restrict ourselves to the friction force $\overrightarrow{F_{\text{fr}}}$ of the board with the water and the propelling force $\overrightarrow{F_{\text{pf}}}$,

$$\overrightarrow{F_{\text{pf}}} + \overrightarrow{F_{\text{fr}}} = \overrightarrow{F_{\text{tot}}} = \overrightarrow{0}, \qquad F_{\text{pf}} - F_{\text{fr}} = F_{\text{tot}} = 0, \tag{2}$$

see, e.g., [3], Kap. 9 and 11. Here and in the following, for a vector $\overrightarrow{w}$ the term $w := \|\overrightarrow{w}\|_2$ denotes its modulus.

To compute the friction, a commonly used approximative formula is (see [2], Chapter 5.10, or [6], Kap. 4)

$$F_{\text{fr}} = \frac{1}{2}\,\varrho_W\, c_w\, A_B\, v^2, \tag{3}$$

where ϱ_W, c_w and A_B are known quantities, namely, the density of the water, the drag coefficient and the engaged area of the board. However, during the numerical experiments reported in Section 3, it turned out that this term alone produces mainly upwind directions which are known not to be the fastest ones. This has motivated us to include an additional term which models the force acting on the fin opposing the leeway so that we extended F_{fr} to

$$F_{\text{fr}} = \frac{1}{2}\,\varrho_W\, \left(c_w\, A_B + \mu\, A_F\, c_{w,F}\, \cos\tfrac{\alpha}{2}\right)\, v^2, \tag{4}$$

where A_F is the size of the fin, $c_{w,F}$ its drag coefficient and μ a leeway parameter.

To determine the propelling force, note that the forces attacking the sail are perpendicular to the surface area of the sail and, therefore, are split as vectors into a part parallel and a part perpendicular to the direction of movement. Since this model does not consider any leeway, we assume that the perpendicular part is compensated completely. For the parallel term, we have

$$F_{\text{pf}} = \sin\beta\, F_S \tag{5}$$

where F_S denotes the total force affecting the sail.

The force F_S is composed essentially of three components which will be modeled next. Its presentation largely follows [4] since there the explanation and graphics appeared most transparent to us. Figure 2 and all subsequent figures in this section are taken from [4] and supplemented with a description of the graphics in English. For simplicity, it is assumed that the front part of the sail is a plane.

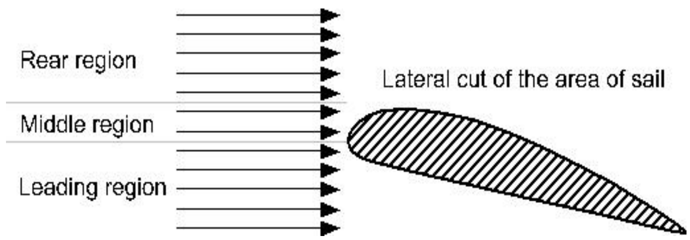

FIGURE 2. Decomposition of air stream near the sail into three regions.

Firstly, air particles colliding with the sail from ahead will be reflected and transfer their momentum onto the sail. We denote this force in the luff of the sail by F_f. Similarly, air particles flowing behind the sail are deviated which also leads to a transfer of momentum onto the sail. The resulting force will be denoted by F_b. In the area in between, the so-called Venturi effect comes into play. It generates a pressure difference between the front and the back side of the sail and results in a further force acting on the sail, denoted by F_m.

We shall now explain these three components in detail. In the area ahead of the sail, the air particles hit the sail at an angle δ, see Figure 3. For simplicity, we have assumed that the Brownian motion of the air particles can be neglected, i.e., all particles hit the sail under the same angle independent of their location. Thus, each particle can be assigned the same momentum $\overrightarrow{p_L}$. This, in turn, splits into one part parallel and one part perpendicular to the sail. Since the mass m_L of an air particle vanishes in relation to the mass of the sail, air particles are reflected under the same angle ('elastic collision'). In doing so, the perpendicular part of the momentum changes its sign while the parallel part remains. Thus, the transfer in momentum onto the sail denoted by Δp_S is

$$\Delta p_S = 2\,\sin\delta\; p_L = 2\,m_L\,W_a\,\sin\delta \tag{6}$$

per air particle which collides with the sail, see [3], Kap. 9 and 10.

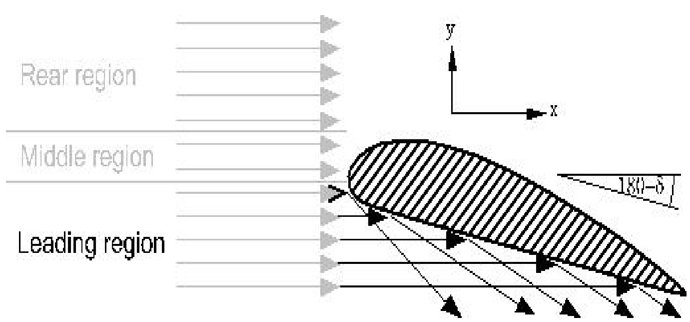

FIGURE 3. Deflection of air particles in region ahead of the sail.

Finally, the force F_S acting on the sail results from the number of particles hitting the sail per time unit, $\frac{\Delta N}{\Delta t}$, multiplied by the transfer of this momentum. The number $\frac{\Delta N}{\Delta t}$, in turn, follows from the density of the particles, $\frac{\Delta N}{\Delta V}$, the speed of the apparent wind and the engaged area of the sail as

$$\frac{\Delta N}{\Delta t} = \frac{\Delta N}{\Delta V} W_a A_S \sin\delta \tag{7}$$

where A_S denotes the surface area of the sail. Since the density of the air is defined as

$$\varrho_L = m_L \frac{\Delta N}{\Delta V}, \tag{8}$$

we arrive at a description of the force ahead of the sail in the form

$$F_f = 2\,\varrho_L\, A_S\, W_a^2 \sin^2\delta. \tag{9}$$

Next we determine (an approximation of) the force F_b behind the sail. Here also air particles are deviated from their original direction which enforces a transfer of momentum to the sail. This deviation does not only result, like in the previous case, in a direct collision. If the particles continued to flow in a straightforward manner, this would result in a vacuum behind the sail, see Figure 4. In reality, however, a slight under-pressure is generated by which the trajectories of the air particles bend towards the area behind the sail's surface.

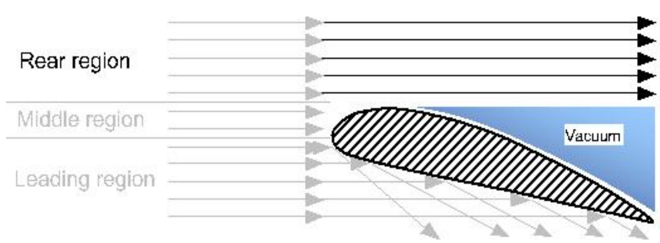

FIGURE 4. Vacuum in region behind sail.

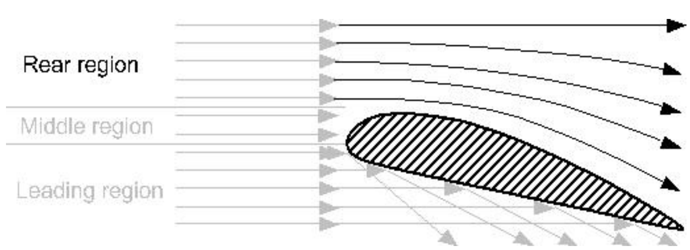

FIGURE 5. Deviation of air particles in area behind sail.

Since here the transfer of momentum of the particles is at most half as large as the one generated by the reflexion at the luff of the sail, we approximate this force by

$$F_b = \eta\, F_f, \tag{10}$$

where $0 \le \eta \le \frac{1}{2}$ denotes a given pressure force parameter.

In the regions directly at the sail, the Venturi effect is most noticeable. It follows from Bernoulli's equation for gas dynamics

$$\frac{1}{2}\,\varrho\, v^2 + \varrho\, g\, H + p = \text{const} \tag{11}$$

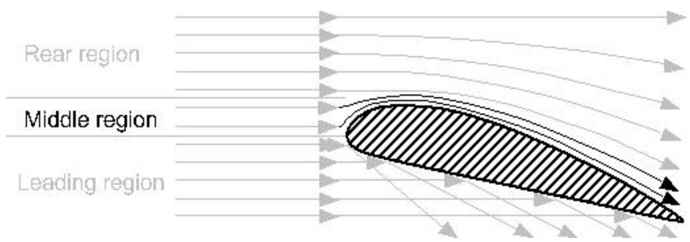

FIGURE 6. Motion of air particles in middle region of sail.

(employing ϱ and p for the gas' density and pressure, H for the height above the water surface and g for the gravity) which is a simplified model for the Navier-Stokes' equation for incompressible and non-viscous fluids or gases. Since the air stream on the luff side appears at the same height as on the lee side and later cancels out, we can neglect the second term and arrive at

$$\frac{1}{2}\,\varrho_L\,v_f^2 + p_f = \frac{1}{2}\,\varrho_L\,v_b^2 + p_b \tag{12}$$

where $\overrightarrow{v_f}$ is the speed of air ahead of the sail (in luff), $\overrightarrow{v_b}$ is the speed of air behind the sail (in lee) and p_f, p_b denote the pressures ahead and behind the sail, respectively. If now the air in lee moves faster than in luff, this results in a difference of pressure, which, in turn, generates some force onto the sail. The particles must move faster behind the sail because of the larger path length there, see Figure 6.
One imagines that in the Brownian motion of the particles in lee a preferred direction of movement is induced parallel to the sail, while their average speed remains the same. Consequently, the particles collide less with the sail and, effectively, the air pressure decreases, see Figure 7 and [6], Kap. 1.5, as well as [2], Kap. 1.7.

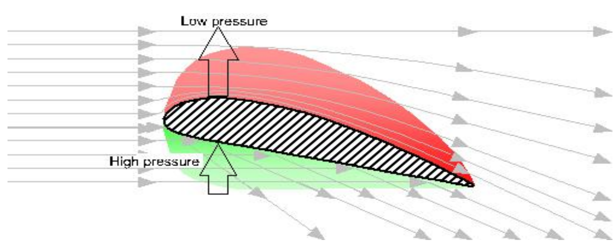

FIGURE 7. Pressure distribution in middle region of sail.

Based on the above reasoning, the velocity of the air particles behind the sail is the λ-fold of the velocity ahead of the sail where $\lambda > 1$ denotes a given parameter for describing the Venturi effect. Thus, we can conclude for the pressure difference

$$p_f - p_b = \frac{1}{2}\,\varrho_L\,(v_b^2 - v_f^2) = \frac{1}{2}\,\varrho_L(\lambda^2 - 1)\,v_f^2 \tag{13}$$

and, employing $v_f \approx -\cos\delta\, W_a$, the force in the middle area of the sail results in

$$F_m = \frac{\lambda^2 - 1}{2}\,\varrho_L\,W_a^2\,\cos^2\delta\,A_S. \tag{14}$$

The total force acting on the sail results by combining (9), (10) and (14) as

$$F_S = F_f + F_m + F_b \tag{15}$$

so that the propelling force can be modeled as

$$\begin{aligned} F_{\text{pf}} &= \sin\beta F_S = \sin\beta A_S \varrho_L \left(2(1+\eta)\sin^2\delta + \frac{\lambda^2-1}{2}\cos^2\delta\right) W_a^2 \\ &= \sin\beta A_S \varrho_L \left(2(1+\eta)(1-\cos^2\delta) + \frac{\lambda^2-1}{2}\cos^2\delta\right) W_a^2. \qquad (16) \end{aligned}$$

Below we rewrite this term by using another representation for $\cos\delta$ from Figure 1, namely,

$$\cos\delta = \frac{\overrightarrow{n}\cdot(-\overrightarrow{W_a})}{W_a}, \qquad n = 1. \qquad (17)$$

As described previously, the surfer moves with uniform speed as soon as the forces acting on him cancel each other out. In view of (1) and (2), the equation for the maximal velocity dependent on the direction and the position of the sail is finally of the form

$$\begin{aligned} 0 &= F_{\text{tot}} \\ &= \sin\beta\, A_S\, \varrho_L\, |\overrightarrow{W_o} - \overrightarrow{v}|^2 \\ &\quad \times \left(2(1+\eta)\left(1 - \frac{(\overrightarrow{n}\cdot(\overrightarrow{W_o}-\overrightarrow{v}))^2}{(\overrightarrow{W_o}-\overrightarrow{v})^2}\right) + \frac{\lambda^2-1}{2}\frac{(\overrightarrow{n}\cdot(\overrightarrow{W_o}-\overrightarrow{v}))^2}{(\overrightarrow{W_o}-\overrightarrow{v})^2}\right) \\ &\quad - \frac{1}{2}\varrho_W\, \left(c_w\, A_B + \mu\, A_F\, c_{w,F}\, \cos\tfrac{\alpha}{2}\right) v^2. \qquad (18) \end{aligned}$$

Note that this formulation of the problem is invariant under scaling of the velocities since $\overrightarrow{v}$ and $\overrightarrow{W_o}$ enter quadratically in (18).

In the following, we discuss numerical solution strategies to determine $\overrightarrow{v}$ from (18).

3. Numerical Simulation and Visualization

The algebraic equation (18) is of the form

$$J(\overrightarrow{v}) = 0 \qquad (19)$$

which is scalar but nonlinear in $\overrightarrow{v}$. According to its derivation, its solution $\overrightarrow{v}$, if it exists, provides a maximal speed $\overrightarrow{v_{\max}}$ achievable by the surfer.

One possible solution strategy is to apply Newton's method to (19) which, however, is guaranteed to converge only locally which poses the problem to determine an appropriate initial guess. Globally convergent iterative methods to solve (19) are only of first order and considered too slow for our purpose. In any case, the iterative strategy described below which resorts to applying Newton's second law provides a different way to compute an approximation to $\overrightarrow{v}$ when applied to the situation at hand,

$$\overrightarrow{F_{\text{tot}}} = m\,\overrightarrow{a}, \qquad (20)$$

see, e.g., [3], Kap. 9, with acceleration $\overrightarrow{a} = \frac{d}{dt}\overrightarrow{v}$ and force F_{tot} from (18). Because of the scaling invariance of (19) with respect to the velocity, we choose the scale where the original wind speed $\overrightarrow{W_o}$ is one.

In view of the surfer's practical problem addressed in the introduction, to choose the sail of the size optimal for him, we have to solve an optimization problem to maximize A_S over a given finite set of sails of possible area sizes with the dynamics of the process governed by (19) and (20). In addition, $\overrightarrow{v}$ and the angles α and β are unknown so that we are facing a discrete optimization problem subject to a nonlinear differential-algebraic system of equations with additional inequality constraints (due to physical reasons).

Taking into account the numerical challenge addressed at the end of the introduction, to provide a simulation on a common laptop within a few minutes, we have decided on an iterative strategy to not solve the complete optimization problem but a simpler suboptimal one as follows. Each iteration consists of essentially three stages. First, for given angles, we compute an approximate velocity $\overrightarrow{v}$ based on the relation between acceleration and velocity, and on a discretization of (20) in terms of a one-step numerical method to solve ordinary differential equations (ODEs), see, e.g., [5]. We then update the apparent wind $\overrightarrow{W_a}$ according to (1). Third, we compute a new approximation to $F_{\rm pf}$ based on (16) and use this information to update the velocity, and so on.

Our basic routine is therefore the following. We denote by $h > 0$ the step size for the numerical method to solve the ODE (20). Let $\overrightarrow{v}^{(0)}$ be an initial guess for the velocity and let α, β, δ be given angles. For large times or after many iteration steps, the ith iterative $\overrightarrow{v}^{(i)}$ shall approximate the maximal velocity $\overrightarrow{v_{\max}}$. Moreover, in this case we expect $\frac{d}{dt}\overrightarrow{v_{\max}} = 0$ so that we set $m = 1$. The iteration terminates when the Euclidean norm between two subsequent iterates is below tol for some tol > 0 depending on the step size h.

Solve $[\overrightarrow{v}^{(0)}, \overrightarrow{W_a}^{(0)}, h] \to \overrightarrow{v}^{(\text{final})}$

(i) Set $i = 0$.
(ii) Compute $F_{\rm PF}^{(i)} = F_{\rm PF}(\overrightarrow{W_a}^{(i)})$ according to (16).
(iii) Solve the ODE (20) employing $F_{\rm PF}^{(i)}$ by an explicit one-step method with step size h and input $\overrightarrow{v}^{(i)}$. Denote the result by $\overrightarrow{v}^{(i+1)}$.
(iv) If $\|\overrightarrow{v}^{(i+1)} - \overrightarrow{v}^{(i)}\|_2 \le$ tol, set $\overrightarrow{v}^{(\text{final})} := \overrightarrow{v}^{(i+1)}$ and stop.
(v) Set $\overrightarrow{W_a}^{(i+1)} = \overrightarrow{W_a}^{(i)} - \overrightarrow{v}^{(i+1)}$, update the iteration parameter $i = i + 1$ and go to (ii).

Note that the resulting acceleration depends continuously on the velocity and becomes negative for velocities larger than $\overrightarrow{v_{\max}}$. Since, therefore, the acceleration is bounded on $[0, v_{\max}]$, we can always achieve convergence of the method by choosing an ODE solver of sufficiently high order and h small enough. The numerical results provided below employ the classical Runge-Kutta scheme of order four, see, e.g., [5]. We like to point out that in the range we need, the results obtained by an Euler scheme which is only of first order differ only slightly.

For the numerical results, we have chosen the step size as $h = \frac{1}{100}$ and as initial value for the velocity of the surfer $\overrightarrow{v}^{(0)} := \overrightarrow{0}$ and for the apparent wind $\overrightarrow{W_a}^{(0)} = \binom{0}{1}$. Moreover, we have performed different computations for sails of area sizes in m^2 $A_S \in \{5.4, 6.0, 6.6, 7.3, 7.8, 8.3, 9.0, 10.0\}$ (the usual sizes for racing sails) and with the empirically usually observed parameters of $\varrho_L = 1.184$ and $\varrho_W = 1000.0$. Following [7], we have chosen the relatively small value of $c_w = 0.1$ since board and fin are optimized approximately for flows. Moreover, we have taken $A_B = 0.007$ following [1], according to the width of the board (50 cm), the length and thickness of the fin and further board specific parameters. Further parameters used in the program have been optimized following physical principles which we do not describe here in detail; they are $\eta = 0.49$, $\lambda = 1.35$, $A_F = 0.035$ and $c_{w,F} = 1.1$. The program was written in `C++`, see [9] for a printout. All computations were performed with a 1.0 GHz Intel Pentium III processor with 256 MB memory; each run needed

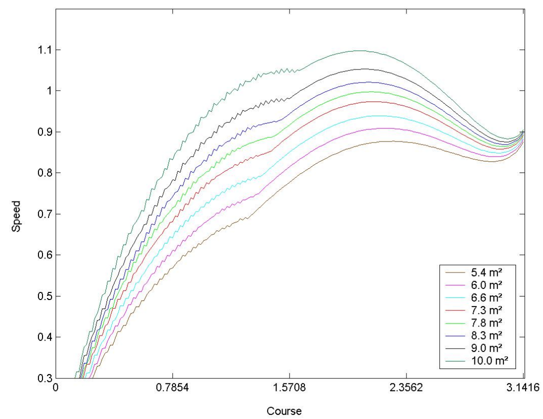

FIGURE 8. Maximal velocity depending on the direction with optimal angle of the sail for sail sizes between 5.4 and 10.0 m^2.

in average 60–300 seconds to converge, depending on the given wind speed and the weight of the surfer.

We have computed the final velocities of the surfer $\overrightarrow{v}^{(\text{final})}$ for different angles. The optimal angles α and β can then be derived from comparing different $\overrightarrow{v}^{(\text{final})}$'s. The results are presented in Figures 8 through 10.

In Figure 8 the maximal velocity depending on the direction with optimal angle β of the sail for sail sizes between 5.4 and 10.0 m^2 are plotted. As expected, the maximal speed is achieved in downwind direction. We would like to point out that the optimal direction for larger sail sizes tends towards smaller angles and, therefore, closer towards the original wind direction. This is closely related to the observation reported by speed windsurfers that higher speed caused by larger sail sizes is obtained in a direction closer to $\overrightarrow{W_o}$. Note further the shape of the functions and, in particular, the qualitative change in the monotonicity in the middle of the diagram, which appears in all cases in the region between half wind and downward wind direction. In addition, we like to point out the increase in velocities for the downward direction. A possible explanation is that in this direction the amount of Venturi's effect decreases strongly while the effect of the pressure force only reaches its maximal value for very large angles. We have computed the solutions to (20) with different step sizes h ranging from 10^{-2} to 10^{-4} and a fourth order as well as a first order method and obtained similar slight oscillations visible in the left half of the graphics which may be explained by the effect the nonlinearity in (18) causes in the iterative solution process.

Figure 9 contains as an example the iterative behavior of the velocities computed with our basic algorithm for a sail of size 8.3 m^2 and different angles. One clearly observes convergence in all cases after a moderate amount of iteration steps.

Finally, Figure 10 illustrates how the maximal velocity depends on the angles α and β for a sail size of $8.3\,m^2$.

4. Validation of Results and Discussion

Although our model is admittedly very simple, the simulations obtained here provide already a remarkably close result when compared to the empirical data for the world record. For an original wind speed of 45 kn, a surfer's weight of 117 kg

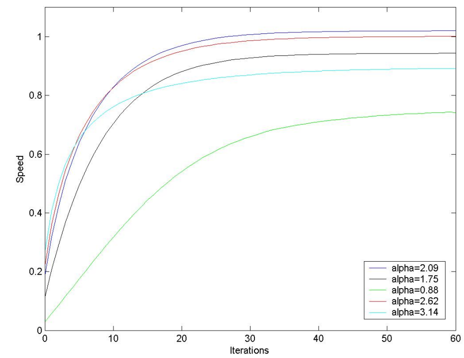

FIGURE 9. Iteratively computed velocities for different directions (in radian).

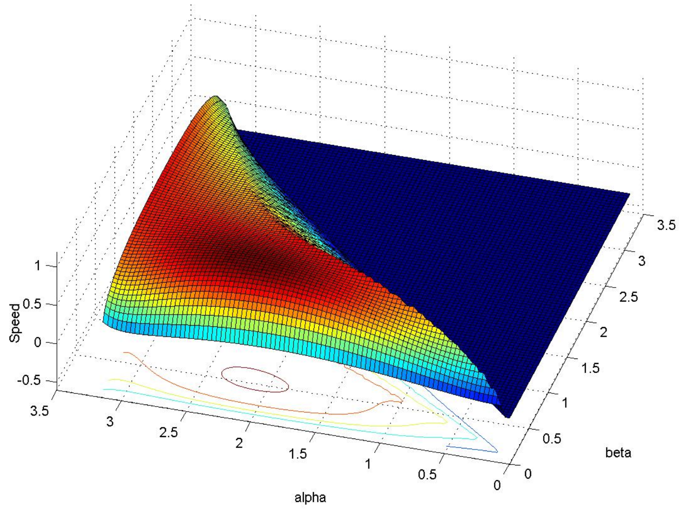

FIGURE 10. Dependence of velocity of α and β for A_S =8.3 m^2.

and a sail with area 5.0 m^2, our program computes a maximal velocity of 44.0137 kn in direction with angle 124°. Two of the parameters were slightly modified to $A_B = 0.004$ and $\mu = 0.33$. As mentioned in the introduction, the up-to-date world record under these conditions (with an original wind speed of approximately 40-45 kn) has been achieved with 48.7 kn under an angle of 126 degrees [8].

We have made a number of simplifying assumptions which allowed the computations reported here. Possible improvements are, of course, to consider a 3D model and to include additional parameters like buoyancy. What we have also neglected so far is the fact that windsurfers usually instinctively move closer towards the original wind direction the faster they are. Moreover, friction forces could be modeled much more precisely by including the profile of waves and turbulence. We expect that a detailed description of the model would involve Navier-Stokes' equations for (in)compressible fluids as well as to consider turbulence models. A 3D model would, in particular, include the weight of the surfer, the board and the rig as well as the buoyancy of the board which would then be a function depending on the velocity. This would, in turn, also result in a dependence on the friction with the water via a decrease of the engaged area of the board. In addition, a vertical component for the force in the sail would come into play which results from a slight acute position of the sail.

Of course, any dynamical behavior of the flow can be modeled much more precisely by Navier-Stokes' equations which would have an effect on the description of the dynamics in both media, air and water. In the underwater regime, this would replace the computation of friction through the parameter c_w. In addition, effects like air resistance of board, rig and surfer could be included as well as air vortexes. One would then have to discard the computation of the forces to the sail via the pressure force and Venturi's effect. As the solution of the Navier-Stokes' equations yields velocities but not directly the forces, one would need in this case a new strategy for these. The shape of board, surfer and rig would then also have to be incorporated into the model through the boundary conditions.

Acknowledgments

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