Math 436 Fall 2012 Homework 4 Solutions

Due Nov. 8 in Class

Note. All problem numbers refer to "Updated" version of lecture note.

- **Exercise 3.1.** Consider the Telegrapher's equation

$$u_{xx} = u_{tt} + \lambda u_t \tag{1}$$

(recall that $\lambda > 0$) over the interval $x \in [0, L]$ subject to conditions

$$u(0,t) = u(L,t) = 0; \qquad u(x,0) = f(x), \quad u_t(x,0) = h(x). \tag{2}$$

Use the method of separation of variables to study the limiting behavior of u as $t \longrightarrow \infty$.

- **Exercise 3.5.** Consider the boundary value problem for $u(x,y)$ in the annular region:

$$u_{xx} + u_{yy} = 0 \quad \rho^2 < x^2 + y^2 < 1; \qquad u(x,y) = \begin{cases} f & x^2 + y^2 = \rho^2 \\ g & x^2 + y^2 = 1 \end{cases}. \tag{3}$$

Obtain the formula for the solution using separation of variables.

- **Exercise 3.8.** Consider the equation

$$r^2 R'' + r R' + (\lambda r^2 - n^2) R = 0, \qquad R(0), R'(0) \text{ bounded}, \ R(1) = 0. \tag{4}$$

a) Prove that there are no negative eigenvalues (along the way you will see why the boundedness of $R'(0)$ is needed.)

b) Prove that $\lambda = 0$ is not an eigenvalue.

c) Let $\lambda_k \neq \lambda_l$ be eigenvalues, prove that the eigenfunctions $R_k(r), R_l(r)$ satisfy

$$\int_0^1 R_k(r) R_l(r) r \, \mathrm{d}r = 0. \tag{5}$$

(Hint: For a) and c), write the equation as

$$\frac{1}{r} \left[(r R')' - \frac{n^2}{r} R \right] + \lambda R = 0 \tag{6}$$

first.)