Comments on Homework 6

March 27, 2014

1. Mistakes.

The following are popular mistakes. Try to fully understand why they are wrong.

i. $\sqrt{\cos^2 t \sin^2 t} = \cos t \sin t$. (Hint:[1])

ii. We have, by Green's Theorem

$$\int_{\partial T^{-1}(D)} ....\mathrm{d}u + ...\mathrm{d}v = \int_{T^{-1}(D)} \left|\det \frac{\partial(X,Y)}{\partial(u,v)}\right| \mathrm{d}(u,v). \tag{1}$$

(Hint:[2])

iii. We have by Green's Theorem

$$\int_{\partial D} |f\,\mathrm{d}x + g\,\mathrm{d}y| = \int_D \left|\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right| \mathrm{d}(x,y). \tag{2}$$

(Hint:[3])

2. Exercises.

Some related exercises.

Exercise 1. Show through an example that in general

$$\int_{\partial D} \left|\begin{pmatrix} f \\ g \end{pmatrix} \cdot \boldsymbol{T}\right| \mathrm{d}s = \int_D \left|\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right| \mathrm{d}(x,y) \tag{4}$$

does not hold. Here $\boldsymbol{T}$ is the unit tangent vector. (Hint:[4])

3. Other comments.

- For integration involving spheres, spherical coordinates may or may not be superior to cylindrical coordinates. The safe way is to try a bit of both and see which one gives you simpler integrals.

---

1. Absolute value.

2. There is no absolute value in Green's Theorem.

3. First so far $\int f\,\mathrm{d}x + g\,\mathrm{d}y$ is just a symbol and therefore $\int |f\,\mathrm{d}x + g\,\mathrm{d}y|$ is not defined. If we want to define it, the natural definition should be

$$\int |f\,\mathrm{d}x + g\,\mathrm{d}y| := \int \left|\begin{pmatrix} f \\ g \end{pmatrix} \cdot \boldsymbol{T}\right| \mathrm{d}s \tag{3}$$

where $\boldsymbol{T}$ is the unit tangent vector. But then (2) in general does not hold. See Exercise 1.

4. Take $f = 0, g = x$ and $D = [1-\varepsilon, 1] \times [-1, 1]$. As $\varepsilon \to 0$ the RHS $\longrightarrow 0$ but the LHS does not.